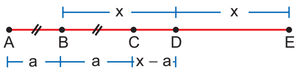
Solucionario Geometría

1.°

APLICAMOS LO APRENDIDO (página 6) Unidad 1

1.



Dato:

$AC + 2CE = 36$
$2a + 2(2x - a) = 36$
$2a + 4x - 2a = 36$
$4x = 36$
$x = 9$

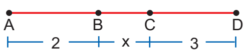
Clave E

2. De la figura planteamos:

$5x - 2 = 10 + 6x - 27$
$15 = x$

Clave D

3.



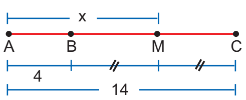
Dato:

$$\frac{1}{AB}+\frac{1}{AD}=\frac{2}{AC}$$
$$\frac{1}{2}+\frac{1}{5+x}=\frac{2}{2+x}$$
$$\frac{x+5+2}{2(5+x)}=\frac{2}{2+x}$$

$20 + 4x = (x + 2)(x + 7)$
$20 + 4x = x^2 + 9x + 14$
$0 = x^2 + 5x - 6$
$0 = (x - 1)(x + 6)$
$\therefore\ x = 1$

Clave C

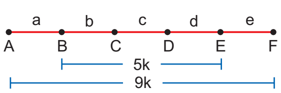
4.



$x = 4 + 5$
$x = 9$

Clave B

5.



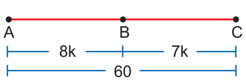
Dato:

$AC + BD + CE + DF = 42$
$(a + b) + (b + c) + (c + d) + (d + e) = 42$
$5k + 5k + 4k = 42$
$14k = 42$
$k = 3$

$\Rightarrow\ BE = 5(3) = 15$

Clave A

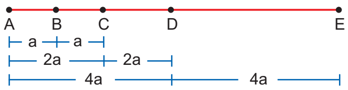
6.



$\Rightarrow 15k = 60$
$k = 4$
$\therefore BC = 7(4) = 28$

Clave C

7.

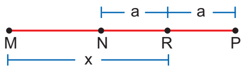


Dato:

$CE - AC = 16$
$6a - 2a = 16$
$4a = 16$
$a = 4$
$\Rightarrow AE = 8(4) = 32$

Clave E

8.

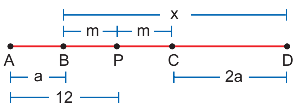


Dato:

$MP + MN = 26$
$(x + a) + (x - a) = 26$
$2x = 26$
$x = 13$

Clave A

9.



$x = 2a + 2m = 2(a + m)$

Pero:

$a + m = 12$
$\Rightarrow x = 24$

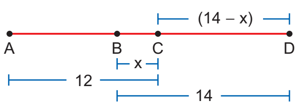
Clave C

10. Del gráfico:

$2x + 17 = 3x - 1 + (4x + 2 - x)$
$2x + 17 = 3x - 1 + 3x + 2$
$16 = 4x$
$x = 4$
$\Rightarrow AC = 3x - 1 = 3(4) - 1 = 11$

Clave D

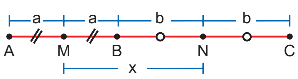
11.



$12 + 14 - x = 24$
$12 - x = 10$
$x = 2$

Clave B

12.



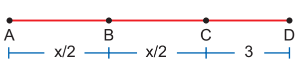
$2a + 2b = 14$
$a + b = x \quad ...(1)$

De (1):

$a + b = 7$
$x = 7$

Clave C

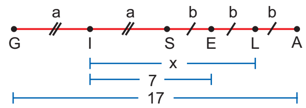
13.



$$\Rightarrow \frac{x}{2}+3=7$$
$x = 8$

Clave E

14.



$\Rightarrow\ a + b = 7\ ;\ a + 2b = x$

$\Rightarrow \underbrace{a+b}_{7} + \underbrace{a+2b}_{x} = 17$
$7 + x = 17$
$x = 10$

Clave B

PRACTIQUEMOS:
Nivel 1 (página 8) Unidad 1

Comunicación matemática

1. VFF

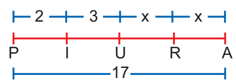
Clave B

2.

Recta L	$\overleftrightarrow{L}$
Rayo AB	$\overrightarrow{AB}$
Segmento CD	$\overline{CD}$

Razonamiento y demostración

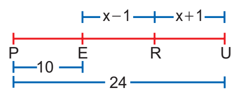
3.



$2 + 3 + x + x = 17$
$2x = 12$
$x = 6$

Clave D

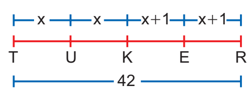
4.



$x - 1 + x + 1 = 14$
$2x = 14$
$x = 7$

Clave B

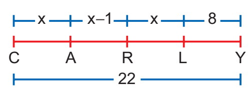
5.



$x + x + x + 1 + x + 1 = 42$
$4x + 2 = 42$
$4x = 40$
$x = 10$

Clave D

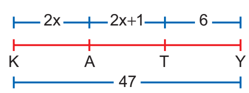
6.



$x + x - 1 + x + 8 = 22$
$3x + 7 = 22$
$3x = 15$
$x = 5$

Clave A

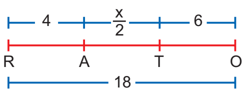
7.



$2x + 2x + 1 + 6 = 47$
$4x = 40$
$x = 10$

Clave A

8.



$4 + \frac{x}{2} + 6 = 18$

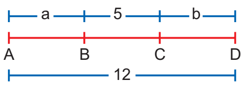
$\frac{x}{2} = 8$

$\therefore x = 16$

Clave E

Resolución de problemas

9.

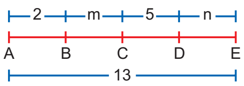


$a + 5 + b = 12$

$a + b = 7$

Clave E

10.



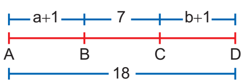
$2 + m + 5 + n = 13$

$m + n + 7 = 13$

$m + n = 6$

Clave A

11.



$a + 1 + 7 + b + 1 = 18$

$a + b = 9$

Clave E

Nivel 2 (página 8) Unidad 1

Comunicación matemática

12. AM $\boxed{=}$ MB

$\overline{AM}$ $\boxed{\cong}$ $\overline{MB}$

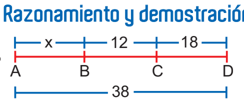
AM $\boxed{=}$ AB/2

13. F V F

Clave E

Razonamiento y demostración

14.



Piden x:

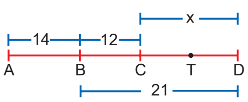
$x + 12 + 18 = 38$

$x + 30 = 38$

$x = 8$

Clave E

15.

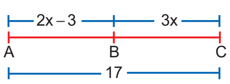


$12 + x = 21$

$x = 9$

Clave A

16.



$2x - 3 + 3x = 17$

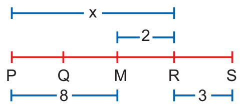
$5x - 3 = 17$

$5x = 20$

$x = 4$

Clave D

17.

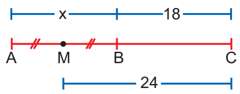


$x = 8 + 2$

$x = 10$

Clave A

18.



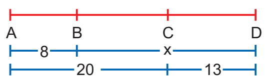
$MB = 24 - 18$

$MB = 6$

$\Rightarrow x = 12$

Clave B

19.

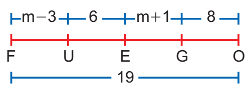


$x = (20 - 8) + 13$

$x = 25$

Clave B

20.



$m - 3 + 6 + m + 1 + 8 = 19$

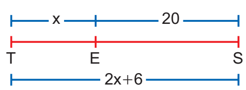
$2m + 12 = 19$

$2m = 7$

$m = 3{,}5$

Clave C

21.

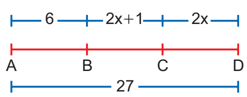


$x + 20 = 2x + 6$

$14 = x$

Clave B

22.



$6 + 2x + 1 + 2x = 27$

$4x + 7 = 27$

$4x = 20$

$x = 5$

$BC = 2x + 1$

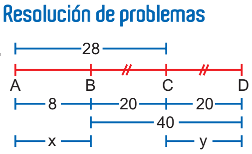
$BC = 2(5) + 1$

$BC = 11$

Clave C

Resolución de problemas

23.

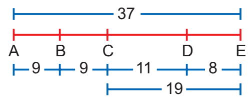


$x = 8 \quad \wedge \quad y = 20$

$\Rightarrow x + y = 8 + 20 = 28$

Clave D

24.

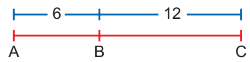


$AB = 9 \qquad CD = 11$

$\Rightarrow AB + CD = 9 + 11 = 20$

Clave B

25.

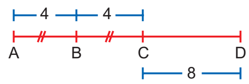


$AC = 12 + 6$

$AC = 18$

Clave C

26.



$BD = 4 + 8$

$BD = 12$

Clave E

Nivel 3 (página 9) Unidad 1

Comunicación matemática

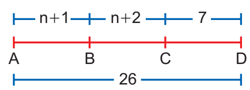
27. F F F

Clave B

28. EF = BC ✓

Razonamiento y demostración

29.



$n + 1 + n + 2 + 7 = 26$

$2n = 16$

$n = 8$

$AC = n + 1 + n + 2$

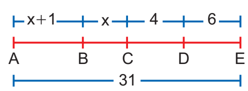
$AC = 2n + 3$

$AC = 2(8) + 3$

$AC = 19$

Clave D

30.



$x + 1 + x + 4 + 6 = 31$

$2x = 20$

$x = 10$

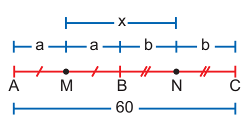
$BD = x + 4$

$BD = 10 + 4$

$BD = 14$

Clave A

31.



$a + b = x$

$a + a + b + b = 60$

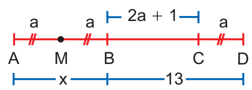
$2a + 2b = 60$

$2(a + b) = 60$

$a + b = 30$

$\Rightarrow x = 30$

Clave D

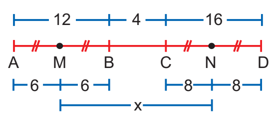
32. 

$2a + 1 + a = 13$
$3a + 1 = 13$
$3a = 13 - 1$
$3a = 12$
$a = \frac{12}{3}$
$a = 4$

Piden AB:
$AB = a + a$
$AB = 4 + 4$
$AB = 8$

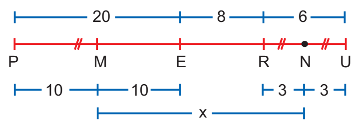
Clave A

Resolución de problemas

33. 

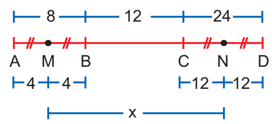
$x = 6 + 4 + 8$
$x = 18$

Clave E

34. 

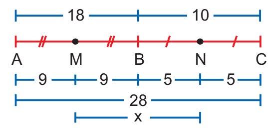
$x = 10 + 8 + 3$
$x = 21$

Clave A

35. 

$x = 4 + 12 + 12$
$x = 28$

Clave B

36. 

$x = 9 + 5$
$x = 14$

Clave E

ÁNGULOS

APLICAMOS LO APRENDIDO (página 11) Unidad 1

1. $2\theta + \theta + 60° = 180°$
$3\theta = 120°$
$\theta = 40°$
$\Rightarrow x = 60° + 40° = 100°$

Clave C

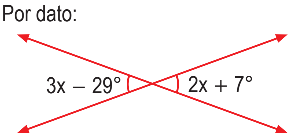
2. Del gráfico:
$10x + 30° + 50° = 180°$
$10x = 100$
$x = 10°$

Clave A

3. Del gráfico:
$x + 180° + 67{,}5° = 360°$
$x = 112{,}5°$
$x = 112°\ 30'$

Clave A

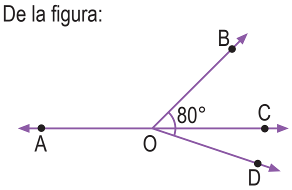
4. Por dato:



$\Rightarrow 2x + 7° = 3x - 29°$
$36° = x$

Clave D

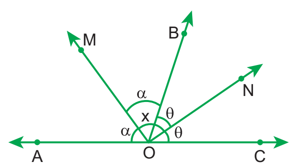
5. De la figura:



$$\begin{array}{r} m\angle AOB + m\angle AOD = 280° \\ m\angle AOD - m\angle AOB = 12° \\ \hline 2m\angle AOD = 292° \end{array} \downarrow (+)$$

$m\angle AOD = 146°$
$m\angle AOB = 134°$
$\therefore m\angle BOC = 46°$

Clave B

6. 

$2\alpha + 2\theta = 180°$
$\alpha + \theta = 90°$
$x = \alpha + \theta$
$\Rightarrow x = 90°$

Clave E

7. $2(90° - x) + 3(180° - x) = 500°$
$180° - 2x + 540° - 3x = 500°$
$220° = 5x$
$x = 44°$

Clave C

8. Por propiedad:
$x = (180° - 130°) + (180° - 160°)$
$x = 50° + 20°$
$x = 70°$

Clave E

9. Por ángulos alternos internos:
$\beta = 3\alpha$

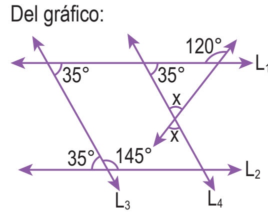
Por propiedad:
$\theta = \alpha + 4\beta$
$\theta = \alpha + 4(3\alpha)$
$\theta = 13\alpha$

Clave A

10. Por propiedad:
$(180° - 140°) + 20° + 50° = 30° + 4x$
$110° = 30° + 4x$
$80° = 4x$
$20° = x$

Clave C

11. Del gráfico:



$x + 35° = 120°$
$x = 85°$

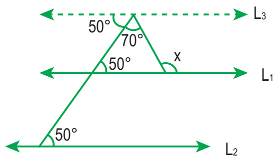
Clave B

12. Por propiedad:
$x = \theta + \alpha$

Por conjugados:
$3\theta + \theta + 3\alpha + \alpha = 180°$
$4(\theta + \alpha) = 180°$
$4x = 180°$
$x = 45°$

Clave B

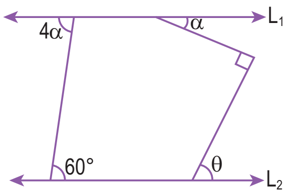
13. Trazamos $\overleftrightarrow{L_3}$ y $\overleftrightarrow{L_1}$



Por ángulos alternos internos:
$x = 70° + 50°$
$x = 120°$

Clave E

14.



Por propiedad:
$\theta + \alpha = 90°$
Por ángulos alternos internos:
$60° = 4\alpha$
$\alpha = 15°$
$\Rightarrow \theta = 90° - 15° = 75°$

Clave C

PRACTIQUEMOS:
Nivel 1 (página 13) Unidad 1

Comunicación matemática

1.
2.
3.

Clave B

Razonamiento y demostración

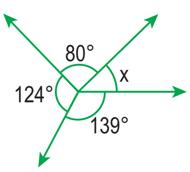
4. $x + x + x = 180°$
$3x = 180°$
$x = 60°$

Clave B

5. $x + 18° + 23° = 70°$
$x = 70° - 41°$
$x = 29°$

Clave B

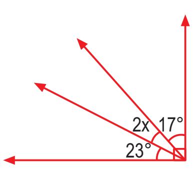
6.



$x + 124° + 80° + 139° = 360°$
$x = 360° - 343°$
$x = 17°$

Clave B

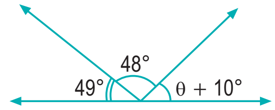
7.



$2x + 17° + 23° = 90°$
$2x + 40° = 90°$
$2x = 50°$
$x = 25°$

Clave D

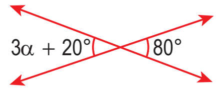
8.



$49° + 48° + \theta + 10° = 180°$
$\theta + 107° = 180°$
$\theta = 180° - 107°$
$\theta = 73°$

Clave E

9.



$3\alpha + 20° = 80°$
$3\alpha = 60°$
$\alpha = 20°$

Clave C

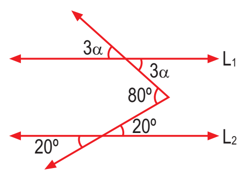
10. $2x = 28°$
$x = 14°$

Clave D

11. $4\alpha = \alpha + 18°$
$3\alpha = 18°$
$\alpha = 6°$

Clave C

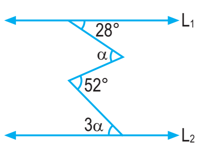
12. Del gráfico:



$3\alpha + 20° = 80°$
$3\alpha = 60°$
$\alpha = 20°$

Clave B

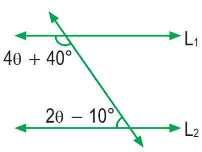
13. Del gráfico:



$3\alpha + \alpha = 28° + 52°$
$4\alpha = 80°$
$\alpha = 20°$

Clave E

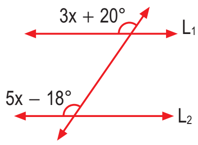
14. Del gráfico:



$4\theta + 40° + 2\theta - 10° = 180°$
$6\theta + 30° = 180°$
$6\theta = 150°$
$\theta = 25°$

Clave B

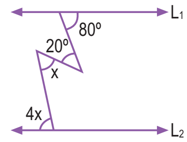
15. Del gráfico:



$3x + 20° = 5x - 18°$
$38° = 2x$
$19° = x$

Clave E

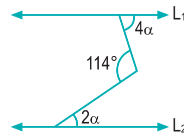
16. Del gráfico:



$80° + x = 20° + 4x$
$60° = 3x$
$20° = x$

Clave A

17. Del gráfico:

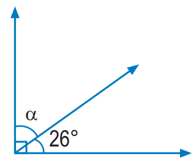


$114° = 2\alpha + 4\alpha$
$114° = 6\alpha$
$19° = \alpha$

Clave C

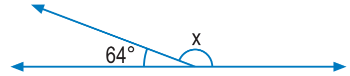
Resolución de problemas

18. Primero hallamos el complemento de 26°:



$\alpha = 90° - 26°$
$\alpha = 64°$

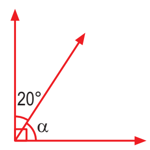
Ahora el suplemento del complemento de 64°:



$x + 64° = 180°$
$x = 180° - 64°$
$x = 116°$

Clave A

19. Primero calculamos el complemento de 20°:



$\alpha + 20° = 90°$
$\alpha = 70°$

Dato:
El suplemento de x es igual al complemento de 20°. Es decir:
$180° - x = 70°$
$180° - 70° = x$
$110° = x$

Clave D

20. Dato:
El complemento de θ más el suplemento de θ es 150°. Es decir:
$180° - \theta + 90° - \theta = 150°$
$270° - 2\theta = 150°$
$120° = 2\theta$
$60° = \theta$

Clave C

21. $S_{(137°)} = 180° - 137°$
$S_{(137°)} = 43°$
Ahora
$C_{(43°)} = 90° - 43°$
$C_{(43°)} = 47°$

Clave B

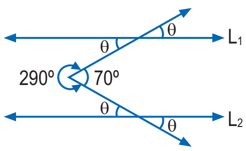
22. $\alpha - (180° - \alpha) = 40°$
$2\alpha - 180° = 40°$
$\alpha = 110°$

Clave E

23. $\alpha + (90° - \alpha) + (180° - \alpha) = 240°$
$-\alpha + 270 = 240°$
$\alpha = 30°$

Clave B

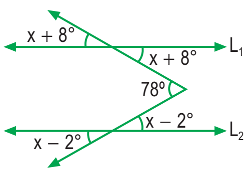
24. Del gráfico:



$\theta + \theta = 360° - 290°$
$2\theta = 70° \Rightarrow \theta = 35°$

Clave A

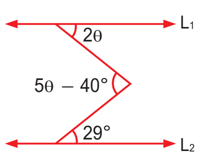
25. Del gráfico:



$x + 8° + x - 2° = 78°$
$2x + 6° = 78°$
$2x = 72° \Rightarrow x = 36°$

Clave C

26. Del gráfico:



$2\theta + 29° = 5\theta - 40°$
$29° + 40° = 5\theta - 2\theta$
$69° = 3\theta \Rightarrow 23° = \theta$

Clave D

27. $4x = x + 57°$
$3x = 57° \Rightarrow x = 19°$

Clave C

Nivel 2 (página 14) Unidad 1

Comunicación matemática

28.

29.

30.

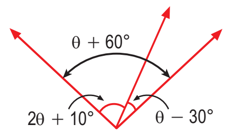
Clave D

31. $180° - 4x = 90° - 2x$
$90° = -2x + 4x$
$90° = 2x$
$45° = x$

Clave C

Razonamiento y demostración

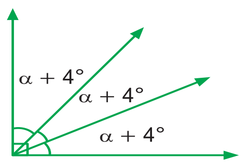
32.



$2\theta + 10° + \theta - 30° = \theta + 60°$
$3\theta - 20° = \theta + 60°$
$2\theta = 80° \Rightarrow \theta = 40°$

Clave C

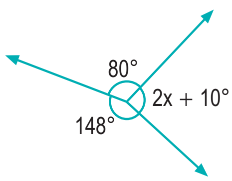
33.



$\alpha + 4° + \alpha + 4° + \alpha + 4° = 90°$
$3\alpha + 12° = 90°$
$3\alpha = 78°$
$\alpha = 26°$

Clave B

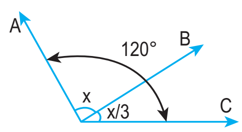
34.



$2x + 148° + 80° + 10° = 360°$
$2x + 238° = 360°$
$2x = 122°$
$x = 61°$

Clave B

35.

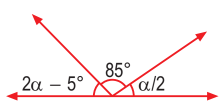


$3\left(x + \frac{x}{3}\right) = (120°)3$
$3x + x = 360°$
$x = 90°$

Piden:
$\frac{x}{3} = \frac{90°}{3} = 30°$

Clave E

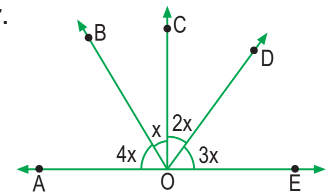
36.



$2\alpha - 5 + \frac{\alpha}{2} + 85° = 180°$
$2\left(2\alpha + \frac{\alpha}{2}\right) = (100°)2$
$4\alpha + \alpha = 200°$
$5\alpha = 200°$
$\alpha = 40°$

Clave C

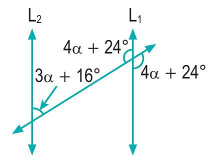
37.



Del gráfico:
$10x = 180°$
$x = 18°$
$m\angle AOB = 4x = 72°$

Clave E

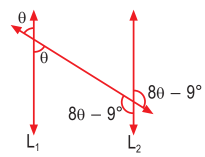
38.



$3\alpha + 16° + 4\alpha + 24° = 180°$
$7\alpha = 140°$
$\alpha = 20°$

Clave E

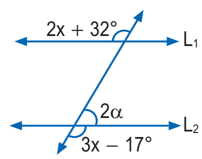
39.



$\theta + 8\theta - 9° = 180°$
$9\theta = 189°$
$\theta = 21°$

Clave D

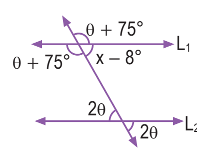
40.



$2x + 32° = 3x - 17°$
$49° = x$
$180° = 3x - 17° + 2\alpha$
$180° + 17° - 3(49°) = 2\alpha$
$\alpha = 25°$

Clave B

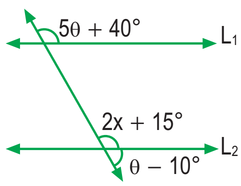
41.



$2\theta + \theta + 75° = 180°$
$3\theta = 105°$
$\theta = 35°$
$35° + 75° + x - 8° = 180°$
$110° + x - 8° = 180°$
$102° + x = 180°$
$x = 78°$

Clave E

42.



$5\theta + 40° + \theta - 10° = 180°$
$6\theta = 150°$
$\theta = 25°$

$2x + 15° + 25° - 10° = 180°$
$2x + 30° = 180°$
$2x = 150°$
$x = 75°$

Clave B

Resolución de problemas

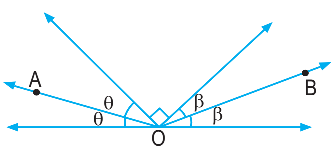
43. $C_{(68°)} = 90° - 68°$
$C_{(68°)} = 22°$
$S_{(22°)} = 180° - 22°$
$S_{(22°)} = 158°$
$S_{(158°)} = 180° - 158°$
$S_{(158°)} = 22°$
$\therefore$ SSC de $68° = 22°$

Clave E

44. $180° - x + 90° - x = 170°$
$270° - 2x = 170°$
$100° = 2x$
$50° = x$
Piden $C_{(x)}$:
$\Rightarrow C_{(x)} = 90° - 50°$
$C_{(x)} = 40°$

Clave A

45.



$m\angle AOB = \theta + \beta + 90°$ ...(1)
Del gráfico:
$2\theta + 2\beta + 90° = 180°$
$\theta + \beta = 45°$
En (1):
$m\angle AOB = 135°$

Clave C

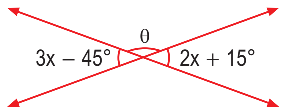
46. $C_{(\alpha)} = \frac{2}{5}S_{(\alpha)}$
$90° - \alpha = \frac{2}{5}(180° - \alpha)$
$450° - 5\alpha = 360° - 2\alpha$
$\alpha = 30°$

Clave B

47. $S_{(\alpha)} - 2C_{(\alpha)} = 40°$
$180° - \alpha - 2(90° - \alpha) = 40°$
$\Rightarrow \alpha = 40°$

Clave D

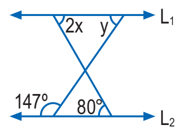
48.



Del gráfico:
$3x - 45° = 2x + 15°$
$x = 60°$
Luego:
$\theta + 2x + 15° = 180°$
$\theta = 45°$
$\therefore C_{(\theta)} = C_{(45°)} = 90° - 45°$
$C_{(45°)} = 45°$

Clave D

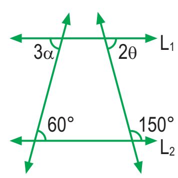
49.



$2x = 80°$
$x = 40°$
$y + 147° = 180°$
$y = 33°$
$x + y = 73°$

Clave D

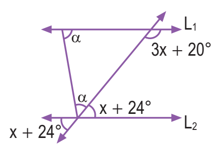
50.



$3\alpha = 60°$
$\alpha = 20°$
$2\theta + 150° = 180°$
$2\theta = 30°$
$\theta = 15°$
$\Rightarrow \theta + \alpha = 35°$

Clave D

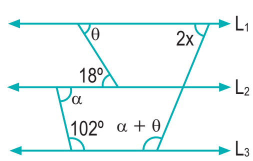
51.



$3x + 20° + x + 24° = 180°$
$4x + 44° = 180°$
$x = 34°$
$2\alpha = 3x + 20°$
$2\alpha = 3(34°) + 20°$
$\alpha = 61°$

Clave C

52.



$\theta = 18°$
$\alpha + 102° = 180°$
$\alpha = 78°$
$2x + \alpha + \theta = 180°$
$2x + 96° = 180°$
$2x = 84°$
$x = 42°$

Clave A

Nivel 3 (página 16) Unidad 1

Comunicación matemática

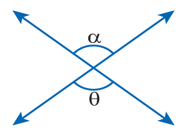
53.

54. $\alpha - (180° - \alpha) = 4(90° - \alpha)$
$2\alpha - 180° = 360° - 4\alpha$
$6\alpha = 540°$
$\alpha = 90°$

Clave B

Razonamiento y demostración

55.

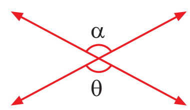


$\alpha = \theta$
Dato:
$\alpha = 3x - 20°$
$\theta = 2x + 10°$
$3x - 20° = 2x + 10°$
$x = 10° + 20°$
$x = 30°$

Piden:
$\alpha = 3x - 20°$
$\alpha = 3(30) - 20°$
$\alpha = 90° - 20°$
$\alpha = 70°$

Clave D

56.

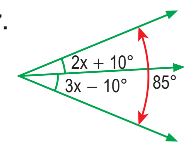


$\alpha = \theta$
Dato:
$\alpha = 6x - 40°$
$\theta = 2x + 20°$
$6x - 40° = 2x + 20°$
$4x = 60°$
$x = 15°$

Piden: θ
$\theta = 2x + 20°$
$\theta = 2(15°) + 20°$
$\theta = 30° + 20°$
$\theta = 50°$

Clave E

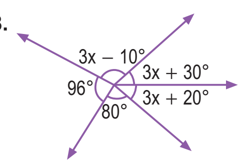
57.



$2x + 10° + 3x - 10° = 85°$
$5x = 85°$
$x = 17°$

Clave B

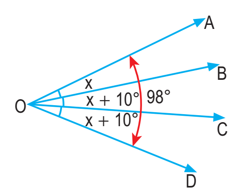
58.



$96° + 80° + 3x - 10° + 3x + 30° + 3x + 20° = 360°$
$9x + 216° = 360°$
$9x = 144°$
$x = 16°$

Clave D

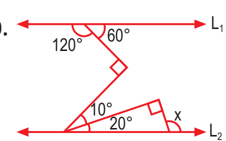
59.



$x + x + 10° + x + 10° = 98°$
$3x + 20° = 98°$
$3x = 78°$
$x = 26°$

Clave D

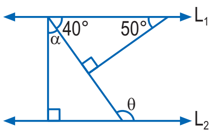
60.



$x = 20° + 90°$
$x = 110°$

Clave B

61.



$\alpha = 90° - 40° = 50°$

$\theta = 90° + \alpha$

$\theta = 90° + 50°$

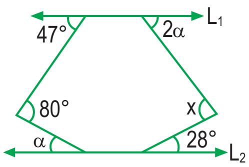
$\theta = 140°$

$\Rightarrow \alpha + \theta = 140° + 50°$

$\alpha + \theta = 190°$

Clave D

62.



$\alpha + 47° = 80°$

$\alpha = 33°$

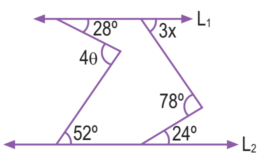
$x = 2\alpha + 28°$

$x = 2(33°) + 28°$

$x = 94°$

Clave E

63.



$3x + 24° = 78°$

$3x = 54°$

$x = 18°$

$28° + 52° = 4\theta$

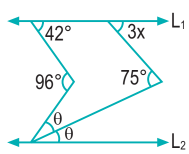
$80° = 4\theta$

$20° = \theta$

$\Rightarrow \frac{\theta}{2} + \frac{x}{2} = 19°$

Clave B

64.



$96° = 42° + 2\theta$

$54° = 2\theta$

$27° = \theta$

$3x + \theta = 75°$

$3x = 75° - 27°$

$3x = 48°$

$x = 16°$

Clave C

Resolución de problemas

65. $2(90° - x) + 3(180° - x) = 400°$

$180° - 2x + 540° - 3x = 400°$

$320° = 5x$

$x = 64°$

Clave C

66. $x + y = 3(70°)$

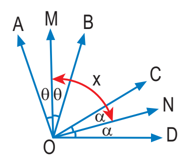
$x - y = 2(90°)$

$\Rightarrow x = \dfrac{3(70°) + 2(90°)}{2}$

$x = 195°$

Clave A

67.



$m\angle AOC = 140°$

$2\theta + x - \theta - \alpha = 140°$

$\theta + x - \alpha = 140° \quad ...(1)$

$m\angle BOD = 80°$

$2\alpha + x - \theta - \alpha = 80°$

$\alpha + x - \theta = 80° \quad ...(2)$

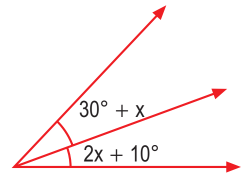
Sumamos (1) y (2):

$2x = 220°$

$x = 110°$

Clave C

68.



$40° + 3x < 90°$

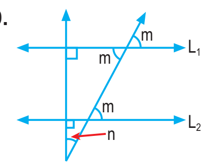
$3x < 50°$

$x < 16{,}6°$

$x_{máx.} = 16°$

Clave B

69.

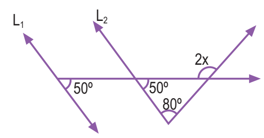


$m + n = 90°$

$\Rightarrow m = 90° - n$

Clave B

70.



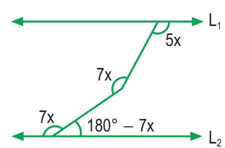
$50° + 80° = 2x$

$130° = 2x$

$x = 65°$

Clave E

71.



$7x = 180° - 7x + 5x$

$7x = 180° - 2x$

$9x = 180°$

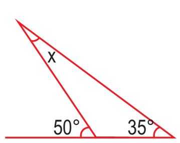
$x = 20°$

Clave A

TRIÁNGULOS

APLICAMOS LO APRENDIDO (página 18) Unidad 1

1.



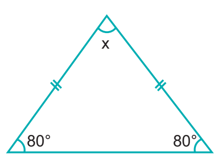
$x + 35° = 50°$

$x = 50° - 35°$

$x = 15°$

Clave D

2.



$80° + 80° + x = 180°$

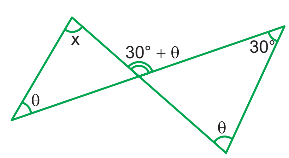
$160° + x = 180°$

$x = 180° - 160°$

$x = 20°$

Clave D

3.

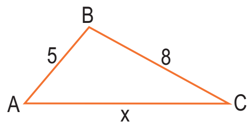


$30° + \theta = \theta + x$

$x = 30°$

Clave E

4.



Por existencia:

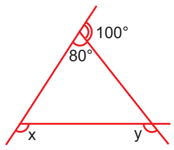
$8 - 5 < x < 8 + 5$

$3 < x < 13$

El máximo valor entero es 12.

Clave B

5.

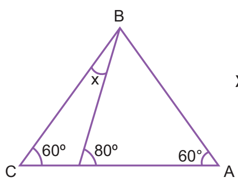


$x + y + 100° = 360°$

$x + y = 260°$

Clave D

6.

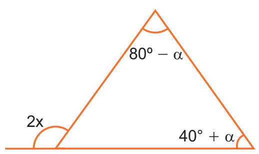


$x + 60° = 80°$

$x = 20°$

Clave B

7.



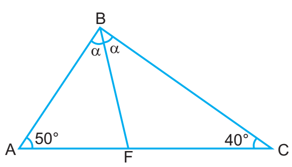
$2x = 80° - \alpha + 40° + \alpha$

$2x = 120°$

$x = 60°$

Clave D

8.



Como $\overline{BF}$ es bisectriz:

$\Rightarrow m\angle ABF = m\angle FBC = \alpha$

Luego En el $\triangle ABC$:

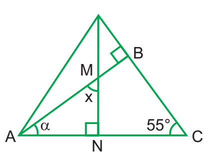
$50° + 40° + 2\alpha = 180°$

$2\alpha = 90°$

$\therefore\ \alpha = 45°$

Clave E

9.



En el ⊿ ABC:

$\alpha + 55° = 90°$

$\alpha = 35°$

En el ⊿ ANM:

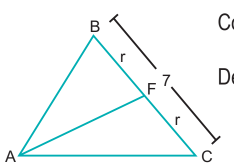
$\alpha + x = 90°$

$35° + x = 90°$

$\therefore\ x = 55°$

Clave C

10.



Como $\overline{AF}$ es mediana:

$\Rightarrow BF = FC = r$

De la figura:

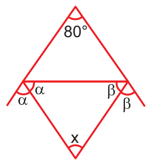
$r + r = 7$

$2r = 7$

$\therefore\ r = 3{,}5$

Clave D

11.



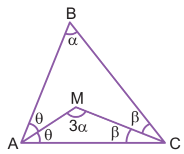
Por propiedad de ángulos formados por bisectrices exteriores:

$x = 90° - \dfrac{80°}{2}$

$x = 50°$

Clave A

12.



Por propiedad de ángulos formados por bisectrices interiores:

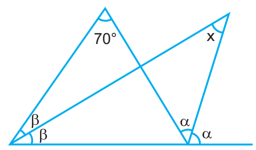
$3\alpha = 90° + \dfrac{\alpha}{2} \quad \Rightarrow \quad 3\alpha - \dfrac{\alpha}{2} = 90°$

$\dfrac{5\alpha}{2} = 90°$

$\therefore \alpha = 36°$

Clave B

13.

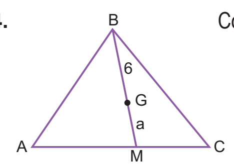


Por propiedad:

$x = \dfrac{70°}{2} \quad \Rightarrow \quad x = 35°$

Clave D

14.



Como G es baricentro:

$\Rightarrow \dfrac{BG}{GM} = \dfrac{2}{1}$

$BG = 2(GM)$

$6 = 2a$

$\therefore\ a = 3$

Clave E

PRACTIQUEMOS:

Nivel 1 (página 20) Unidad 1

Comunicación matemática

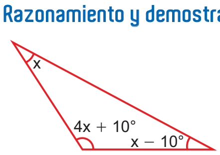
1.

2.

3.

Razonamiento y demostración

4.



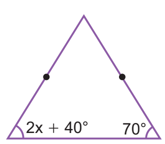
$x + 4x + 10° + x - 10° = 180°$

$6x = 180°$

$x = 30°$

Clave C

5.



Triángulo isósceles:

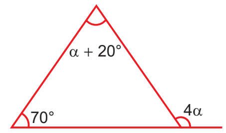
$2x + 40° = 70°$

$2x = 30°$

$x = 15°$

Clave B

6.



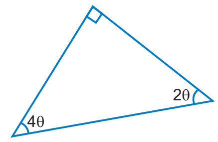
$70° + \alpha + 20° = 4\alpha$

$90° = 3\alpha$

$30° = \alpha$

Clave C

7.

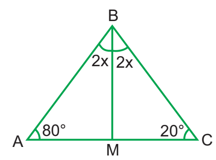


$6\theta = 90°$

$\theta = 15°$

Clave E

8.



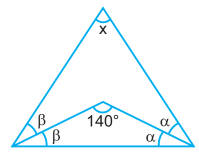
$4x + 100° = 180°$

$4x = 80°$

$x = 20°$

Clave E

9.



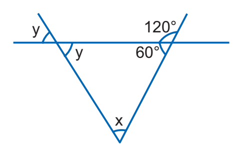
$140° = 90° + \dfrac{x}{2}$

$50° = \dfrac{x}{2}$

$100° = x$

Clave C

10.



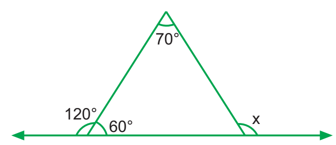
Del gráfico:

$x + y + 60° = 180°$

$\therefore\ x + y = 120°$

Clave B

11.



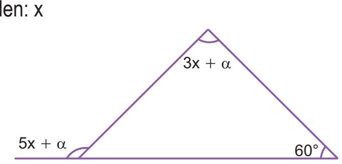
Del gráfico:

$70° + 60° = x$

$\therefore\ x = 130°$

Clave B

12. Piden: x



Del gráfico:

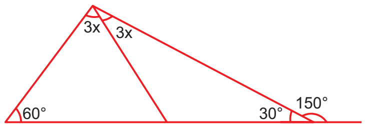
$3x + \alpha + 60° = 5x + \alpha$

$60° = 2x$

$\therefore\ x = 30°$

Clave D

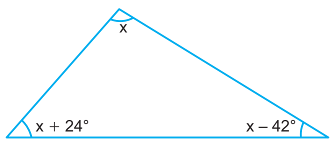
13. Piden: x



Del gráfico:
$60° + 6x + 30° = 180°$
$6x = 90°$
$\therefore x = 15°$

Clave E

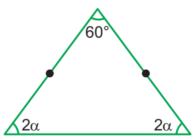
14. Piden: x



Del gráfico:
$x + 24° + x + x - 42° = 180°$
$3x = 198°$
$\therefore x = 66°$

Clave C

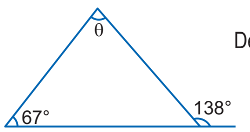
15. Piden: α



Del gráfico:
$60° + 4\alpha = 180°$
$4\alpha = 120°$
$\therefore \alpha = 30°$

Clave D

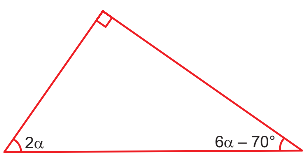
16. Piden: θ



Del gráfico:
$67° + \theta = 138°$
$\therefore \theta = 71°$

Clave B

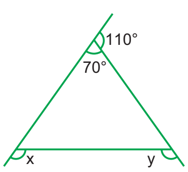
17. Piden: α



Del gráfico:
$2\alpha + 6\alpha - 70° = 90°$
$8\alpha = 160°$
$\therefore \alpha = 20°$

Clave D

18.

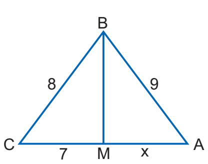


$x + y + 110° = 360°$
$\therefore x + y = 250°$

Clave D

Resolución de problemas

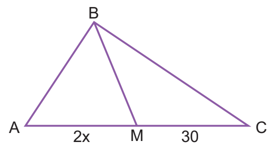
19.



Como $\overline{BM}$ es mediana se cumple:
$CM = MA \Rightarrow 7 = x$

Clave D

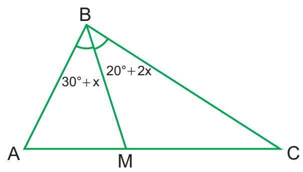
20.



$\overline{BM}$ es mediana, entonces:
$2x = 30$
$\therefore x = 15°$

Clave A

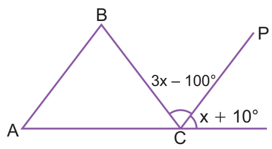
21.



$30° + x = 20° + 2x$
$x = 10°$
$m\angle ABM = 30° + x$
$\therefore m\angle ABM = 40°$

Clave B

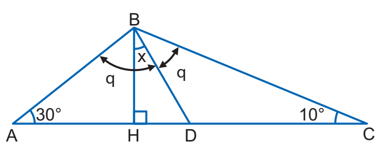
22.



$3x - 100° = x + 10°$
$2x = 110°$
$\therefore x = 55°$

Clave B

23.



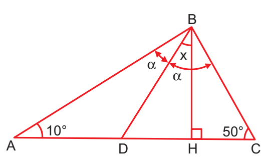
Por propiedad:

$x = \dfrac{30° - 10°}{2} = \dfrac{20°}{2} = 10°$

$\therefore x = 10°$

Clave B

24.



Por propiedad:

$x = \dfrac{50° - 10°}{2} = \dfrac{40°}{2} = 20°$

$\therefore x = 20°$

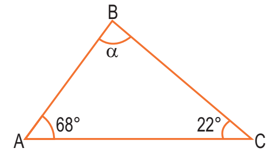
Clave B

Nivel 2 (página 25) Unidad 1

Comunicación matemática

25.

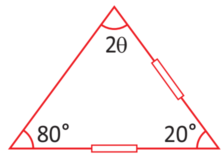
26.



$\alpha + 68° + 22° = 180°$
$\Rightarrow \alpha = 90°$
$\therefore$ El $\triangle$ ABC es un triángulo rectángulo

Clave D

27.

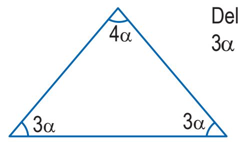


$2\theta + 80° + 20° = 180°$
$2\theta = 80°$
$\Rightarrow \theta = 40°$
$\therefore$ El triángulo es isósceles.

Clave A

Razonamiento y demostración

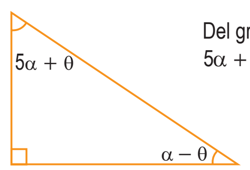
28. Piden: α



Del gráfico:
$3\alpha + 4\alpha + 3\alpha = 180°$
$10\alpha = 180°$
$\therefore \alpha = 18°$

Clave D

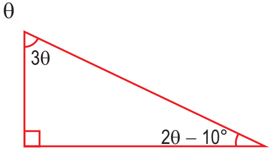
29. Piden: α



Del gráfico:
$5\alpha + \theta + \alpha - \theta = 90°$
$6\alpha = 90°$
$\therefore \alpha = 15°$

Clave D

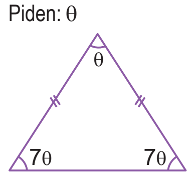
30. Piden: θ



Del gráfico:
$3\theta + 2\theta - 10° = 90°$
$5\theta = 100°$
$\therefore \theta = 20°$

Clave B

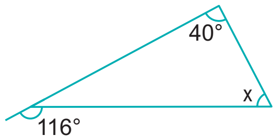
31. Piden: θ



Del gráfico:
$7\theta + 7\theta + \theta = 180°$
$15\theta = 180°$
$\therefore \theta = 12°$

Clave D

32. Piden: x



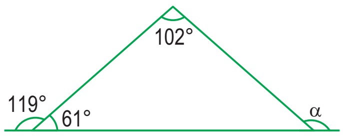
Del gráfico:

$40° + x = 116°$

$\therefore\ x = 76°$

Clave C

33. Piden: α



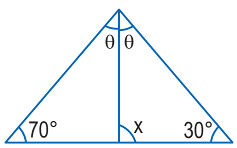
Del gráfico:

$61° + 102° = \alpha$

$\therefore\ \alpha = 163°$

Clave E

34. Piden: x



Del gráfico:

$70° + \theta = x$

$\theta = x - 70°$ .... (1)

También:

$x + \theta + 30° = 180°$

$x + \theta = 150°$ .... (2)

Reemplazando (1) en (2):

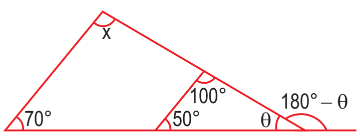
$x + x - 70° = 150°$

$2x = 220°$

$\therefore\ x = 110°$

Clave C

35.



Por ángulo exterior:

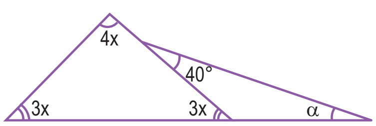
$x + 70° = 100° + 50° = 180° - \theta$

$x + 70° = 150°$

$\therefore\ x = 80°$

Clave E

36.



Por suma de ángulos interiores:

$3x + 4x + 3x = 180°$

$10x = 180°$

$x = 18°$

Por ángulo exterior:

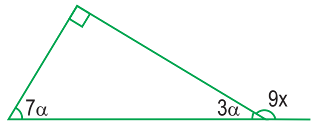
$3x = 40° + \alpha$

$54° = 40° + \alpha$

$\therefore\ \alpha = 14°$

Clave B

37.



Del gráfico:

$7\alpha + 3\alpha = 90° \quad \wedge \quad 3\alpha + 9x = 180°$

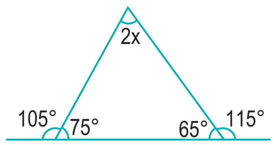
$10\alpha = 90° \qquad 27° + 9x = 180°$

$\alpha = 9° \qquad 9x = 153°$

$\therefore\ x = 17°$

Clave D

38.



Por suma de ángulos interiores:

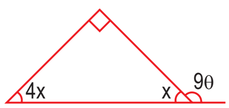
$2x + 75° + 65° = 180°$

$2x = 40°$

$\therefore\ x = 20°$

Clave D

39.



Del gráfico:

$4x + x = 90° \quad \wedge \quad 9\theta + x = 180°$

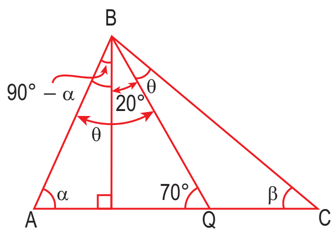
$5x = 90° \qquad 9\theta = 162°$

$\therefore\ x = 18° \qquad \therefore\ \theta = 18°$

Clave A

Resolución de problemas

40. Piden: $\alpha - \beta$



Del gráfico:

$\theta = 90° - \alpha + 20°$

$\theta = 110° - \alpha$ .... (1)

También:

$\theta + \beta = 70°$ .... (2)

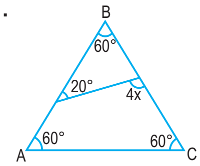
Reemplazando (1) en (2):

$110° - \alpha + \beta = 70°$

$\therefore\ \alpha - \beta = 40°$

Clave A

41.



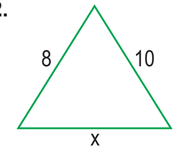
$60° + 20° = 4x$

$80° = 4x$

$20° = x$

Clave D

42.



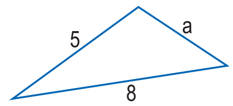
$10 - 8 < x < 10 + 8$

$2 < x < 18$

$\therefore\ x_{máx.} = 17$

Clave D

43.



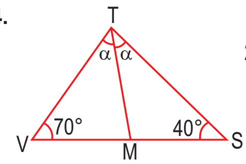
$8 - 5 < a < 8 + 5$

$3 < a < 13$

$\therefore\ a_{mín.} + a_{máx.} = 4 + 12 = 16$

Clave A

44.



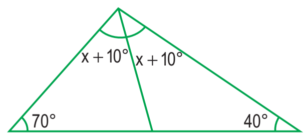
$2\alpha + 110° = 180°$

$2\alpha = 70°$

$\alpha = 35°$

Clave B

45.



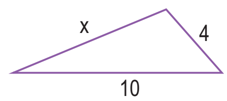
$2x + 20° + 110° = 180°$

$2x = 180° - 130°$

$2x = 50° \quad \Rightarrow \quad x = 25°$

Clave B

46.



$10 - 4 < x < 10 + 4$

$6 < x < 14$

$\therefore\ x_{mín.} = 7$

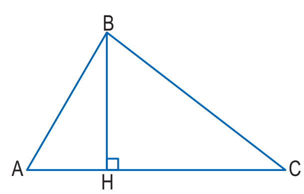
Clave B

Nivel 3 (página 23) Unidad 1

Comunicación matemática

47.

48.



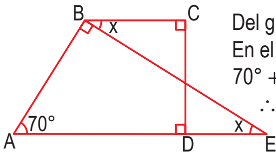
El segmento que parte de un vértice y cae en forma perpendicular al lado opuesto se denomina **altura.**

Clave A

49.

Razonamiento y demostración

50.



Del gráfico: $\overline{BC} \parallel \overline{AE}$

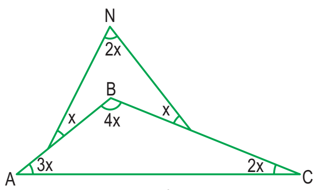
En el ⊿ABE:

$70° + x = 90°$

$\therefore\ x = 20°$

Clave C

51.



Por propiedad: $m\angle B = 4x$

En el △ABC:

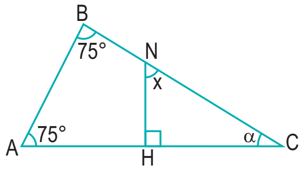
$3x + 4x + 2x = 180°$

$9x = 180°$

$\therefore\ x = 20°$

Clave A

52.



Por dato: AC = BC

Entonces, el △ACB es isósceles, luego:

$75° + 75° + \alpha = 180°$

$\Rightarrow \alpha = 30°$

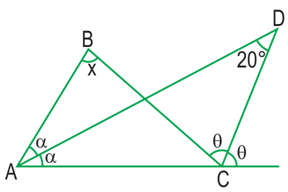
En el ⊿NHC:

$x + \alpha = 90°$

$x + 30° = 90° \quad \Rightarrow\ x = 60°$

Clave E

53.



En el △ABC:

$2\alpha + x = 2\theta \Rightarrow \dfrac{x}{2} = \theta - \alpha \qquad ...(1)$

En el △ADC:

$\alpha + 20° = \theta \Rightarrow 20° = \theta - \alpha \qquad ...(2)$

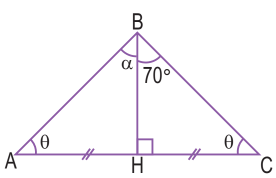
De (1) y (2):

$\dfrac{x}{2} = 20°$

$\therefore\ x = 40°$

Clave A

54.



Para el △ABC: $\overline{BH}$ es altura y mediana (mediatriz), entonces el △ABC es isósceles.

Luego: $70° + \theta = 90° \Rightarrow \theta = 20°$

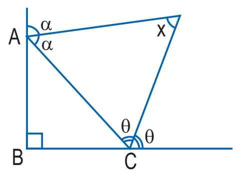
También: $\theta + \alpha = 90°$

$20° + \alpha = 90°$

$\therefore\ a = 70°$

Clave D

55.



Por propiedad:

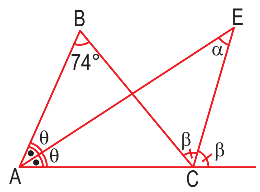
$x = 90° - \dfrac{90°}{2}$

$x = 90° - 45°$

$\therefore\ x = 45°$

Clave B

56.

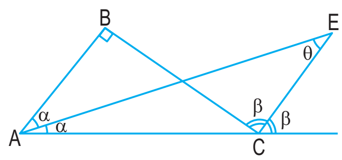


Por propiedad:

$\alpha = \dfrac{74°}{2} = 37° \qquad \therefore\ \alpha = 37°$

Clave B

57.

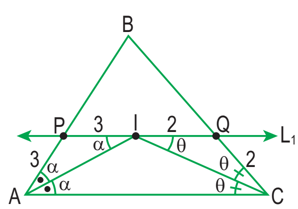


Por propiedad:

$\theta = \dfrac{90°}{2} = 45° \qquad \therefore\ \theta = 45°$

Clave C

58.



Por dato: $\overleftrightarrow{L_1} \parallel \overline{AC}$

Entonces, los triángulos API y CQI resultan isósceles.

Piden:

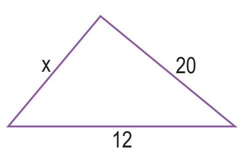
$PQ = PI + IQ = 3 + 2 = 5$

$\therefore\ PQ = 5$

Clave D

Resolución de problemas

59.



Por existencia de un triángulo:

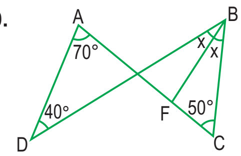
$x < 20 + 12$

$x < 32$

El máximo valor entero de x es 31.

Clave D

60.



Por propiedad:

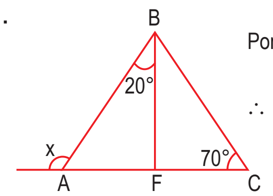
$40° + 70° = 2x + 50°$

$2x = 60°$

$\therefore\ x = 30°$

Clave A

61.



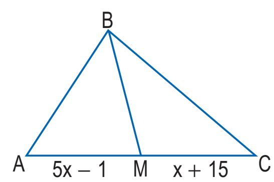
Por ángulo exterior:

$x = 40° + 70°$

$\therefore\ x = 110°$

Clave C

62.



$\overline{BM}$ es mediana, entonces:

$5x - 1 = x + 15$

$4x = 16$

$x = 4$

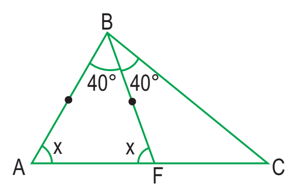
Luego:

$AC = 6x + 14$

$\therefore\ AC = 38$

Clave E

63.



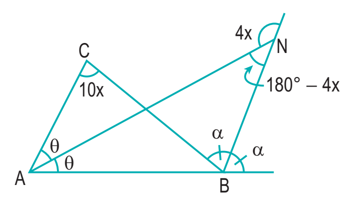
Por suma de ángulos interiores:

$2x + 40° = 180°$

$2x = 140° \ \Rightarrow\ x = 70°$

Clave B

64.



Por propiedad:

$180° - 4x = \dfrac{10x}{2}$

$9x = 180° \ \Rightarrow\ x = 20°$

Clave A

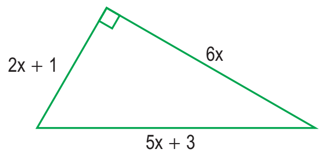
APLICAMOS LO APRENDIDO (página 25) Unidad 1

1. Por el teorema de Pitágoras:

$$125^2 = x^2 + 44^2$$
$$125^2 - 44^2 = x^2$$
$$(125 + 44)(125 - 44) = x^2$$
$$(169)(81) = x^2$$
$$(13)(9) = x$$
$$117 = x$$

Clave B

2.



Por el teorema de Pitágoras:

$$(2x + 1)^2 + (6x)^2 = (5x + 3)^2$$
$$4x^2 + 4x + 1 + 36x^2 = 25x^2 + 30x + 9$$
$$0 = 15x^2 - 26x - 8$$
$$0 = (15x + 4)(x - 2)$$
$$\therefore x = 2$$

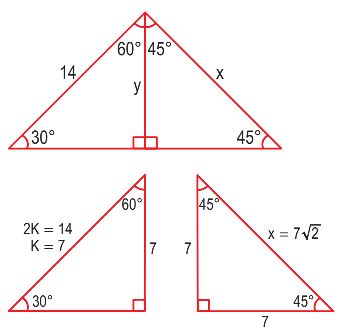
Perímetro = 2x + 1 + 6x + 5x + 3

Perímetro = 2(2) + 1 + 6(2) + 5(2) + 3

Perímetro = 30

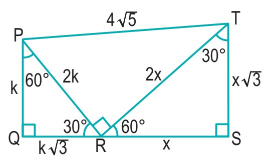
Clave D

3.



Clave E

4.



$$\frac{\text{Perímetro}\triangle PQR}{\text{Perímetro}\triangle RST} = \frac{k(1 + \sqrt{3} + 2)}{x(1 + \sqrt{3} + 2)} = \frac{1}{2}$$

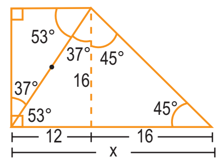
$$2k = x$$

Del $\triangle PRT$:

$$(4\sqrt{5})^2 = (2k)^2 + (2x)^2$$
$$16(5) = 4k^2 + 4x^2$$
$$80 = 5x^2$$
$$x = 4$$

Clave A

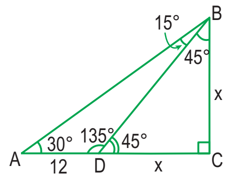
5.



$x = 12 + 16 = 28$

Clave C

6.

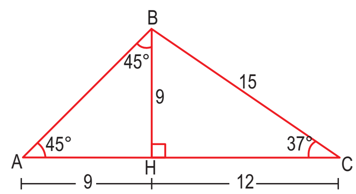


En el ⊿ ACB notable 30° y 60°:

$$12 + x = x\sqrt{3}$$
$$12 = x(\sqrt{3} - 1)$$
$$\Rightarrow x = \frac{12}{\sqrt{3} - 1}$$
$$\therefore x = 6(\sqrt{3} + 1)$$

Clave E

7. Graficamos, luego trazamos la altura BH:



En el ⊿ BHC(37°; 53°):

5k = 15 ⇒ k = 3

Entonces:

HC = 4k = 4(3) = 12

BH = 3k = 3(3) = 9

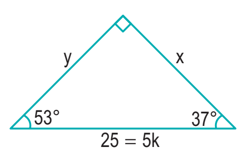
En el ⊿ AHB(45°; 45°):

Si BH = 9 ⇒ AH = 9

Por lo tanto, AC = 21

Clave A

8.



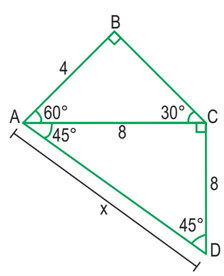
y = 3k ; x = 4k

y = 15 ; x = 20

x − y = 20 − 15 = 5

Clave D

9.



Del ⊿ ACD:

$x = 8\sqrt{2}$

Clave C

10. Como son tres lados, entonces:

x; x + 5; x + 10

Por Pitágoras:

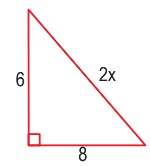
$$x^2 + (x + 5)^2 = (x + 10)^2$$
$$x^2 + x^2 + 10x + 25 = x^2 + 20x + 100$$
$$x^2 - 10x - 75 = 0$$
$$\left.\begin{matrix} x & -15 \\ x & 5 \end{matrix}\right\} \Rightarrow x = 15$$

Por lo tanto, la hipotenusa (lado mayor) es:

x + 10 = 15 + 10 = 25

Clave C

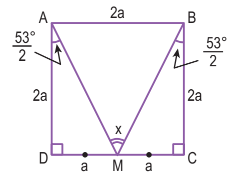
11.



$$(2x)^2 = 6^2 + 8^2$$
$$4x^2 = 36 + 64$$
$$4x^2 = 100$$
$$x^2 = 25$$
$$x = 5$$

Clave A

12.



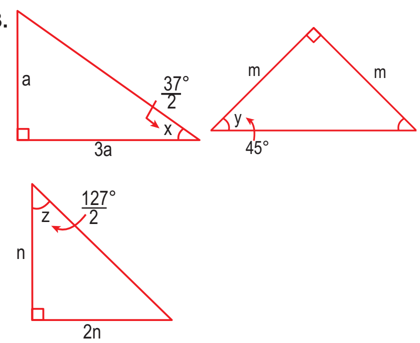
Del gráfico:

$$x = \frac{53°}{2} + \frac{53°}{2}$$

$x = 53°$

Clave B

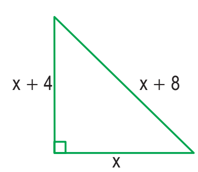
13.



$$\Rightarrow x + y + z = \left(\frac{37°}{2} + 45° + \frac{127°}{2}\right)$$
$$x + y + z = 127°$$

Clave B

14.



$$x^2 + (x + 4)^2 = (x + 8)^2$$
$$x^2 + x^2 + 8x + 16 = x^2 + 16x + 64$$
$$x^2 + 8x + 16 = 16x + 64$$
$$x^2 - 8x = 48$$
$$x(x - 8) = 48$$
$$\Rightarrow x = 12$$
$$x + 8 = 12 + 8 = 20$$

Clave E

PRACTIQUEMOS:

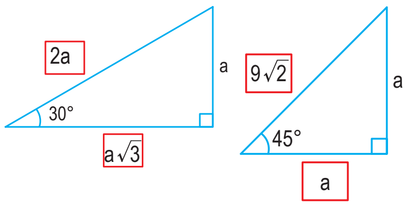
Nivel 1 (página 27) Unidad 1

Comunicación matemática

1. $\triangle$ ABC

Clave D

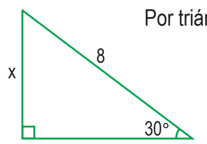
2.



3. I. (II)
II. (I)
III. (III)

Razonamiento y demostración

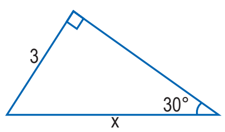
4.



Por triángulo notable sabemos:
$2k = 8$
$k = 4$
Piden x:
$x = k$
$x = 4$

Clave E

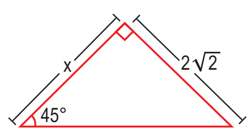
5.



Por triángulo notable de 30° y 60°:
$k = 3$
Piden x:
$x = 2k$
$x = 6$

Clave B

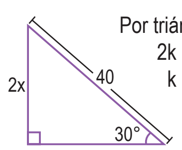
6.



Por triángulo notable de 45°:
$k = 2\sqrt{2}$
Piden x:
$x = k \quad \Rightarrow x = 2\sqrt{2}$

Clave C

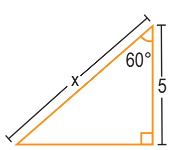
7.



Por triángulo notable de 30° y 60°:
$2k = 40$
$k = 20$
Piden x:
$2x = k$
$2x = 20 \quad \Rightarrow x = 10$

Clave E

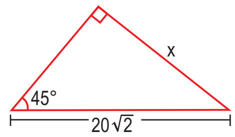
8.



Por triángulo notable de 30° y 60°:
$k = 5$
Piden x:
$x = 2k \quad \Rightarrow x = 10$

Clave C

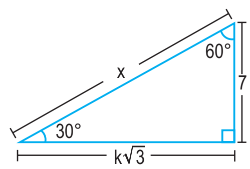
9.



Por triángulo notable de 45°:
$k\sqrt{2} = 20\sqrt{2}$
$k = 20$
Piden x:
$x = k \quad \Rightarrow \quad x = 20$

Clave A

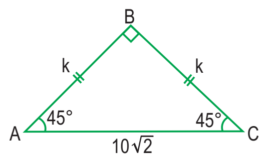
10.



$x = 2k$
$x = 2(7)$
$x = 14$

Clave D

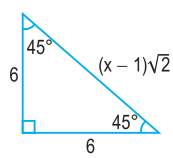
11.



Por triángulo notable:
$k\sqrt{2} = 10\sqrt{2}$
$k = 10$
Piden AB:
$AB = 10$

Clave D

12.

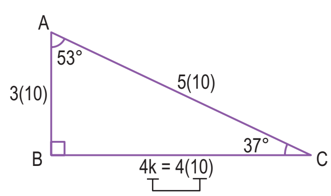


Por triángulo notable:
$k = 6$
Piden x:
$(x-1)\sqrt{2} = k\sqrt{2}$
$x - 1 = 6$
$x = 7$

Clave B

Resolución de problemas

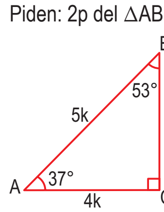
13. Piden: AB + AC



Del gráfico:
$\therefore \ AB + AC = 30 + 50 = 80$

Clave E

14. Piden: 2p del $\triangle$ABC



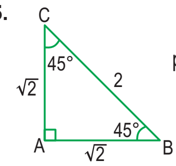
$4m = 3k \Rightarrow k = \frac{4m}{3}$

$\Rightarrow 2p = 12k = 12\left(\frac{4m}{3}\right)$

$\therefore 2p = 16$ m

Clave C

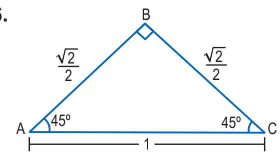
15.



$p_{\triangle CAB} = \frac{2\sqrt{2} + 2}{2} = \sqrt{2} + 1$

Clave C

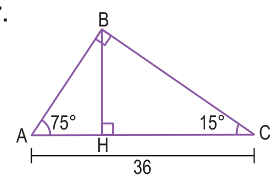
16.



$2p_{\triangle ABC} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + 1 = (\sqrt{2} + 1)$

Clave A

17.



Propiedad:
$BH = \frac{AC}{4}$

$BH = \frac{36}{4} = 9$

$\therefore$ BH = 9 cm

Clave C

Nivel 2 (página 28) Unidad 1

Comunicación matemática

18.

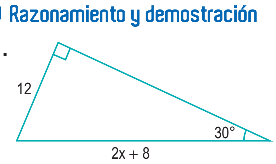
19.

Clave D

20.

Razonamiento y demostración

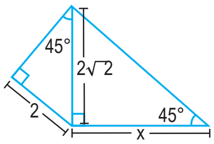
21.



Por triángulo notable:
$k = 12$
Piden x:
$2x + 8 = 2k$
$2x + 8 = 24$
$2x = 24 - 8$
$2x = 16 \quad \Rightarrow \quad x = 8$

Clave C

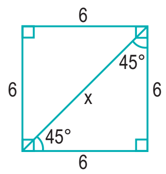
22.



Por triángulo notable:
$k = 2\sqrt{2}$
Piden x:
$x = k$
$x = 2\sqrt{2}$

Clave A

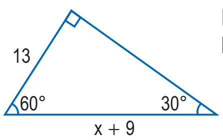
23.



Por triángulo notable:
$k = 6$
Piden x:
$x = k\sqrt{2}$
$x = 6\sqrt{2}$

Clave D

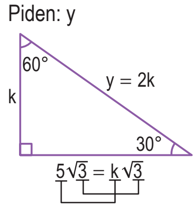
24.



Por triángulo notable:
$k = 13$
Piden x:
$x + 9 = 2k$
$x = 26 - 9$
$x = 17$

Clave B

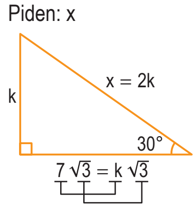
25. Piden: y



$\therefore y = 2(5) = 10$

Clave D

26. Piden: x

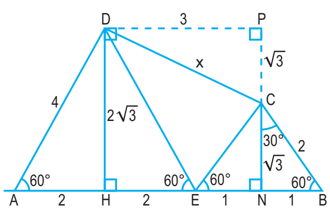


$\therefore x = 2k = 2(7) = 14$

Clave B

Resolución de problemas

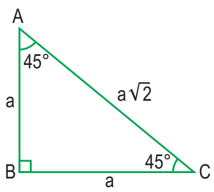
27.



Piden: CD = x
En el ⊿DPC, por el teorema de Pitágoras:
$x^2 = (3)^2 + (\sqrt{3})^2$
$x^2 = 9 + 3 = 12$
$\therefore x = 2\sqrt{3}$

Clave C

28.

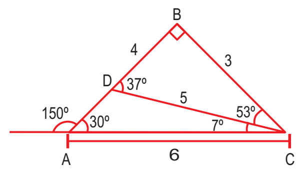


Se tiene el ⊿ABC isósceles.
Piden:

$$\frac{BC}{AC} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Clave B

29.



$2p_{\triangle DBC} = 3 + 4 + 5 = 12$ m

Clave A

Nivel 3 (página 29) Unidad 1

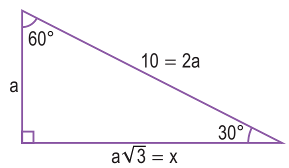
Comunicación matemática

30.

31.

Razonamiento y demostración

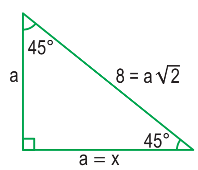
32.



Del gráfico:
$2a = 10 \Rightarrow a = 5$
$x = a\sqrt{3} = (5)\sqrt{3}$
$\therefore x = 5\sqrt{3}$

Clave B

33.

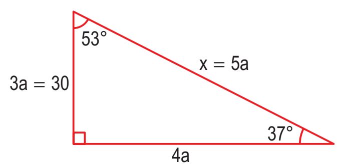


Del gráfico:
$a\sqrt{2} = 8 \Rightarrow a = 4\sqrt{2}$
$x = a = 4\sqrt{2}$

Piden:
$x + 2\sqrt{2} = 4\sqrt{2} + 2\sqrt{2} = 6\sqrt{2}$

Clave C

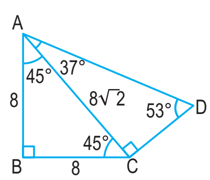
34.



Del gráfico:
$3a = 30 \Rightarrow a = 10$
$x = 5a = 5(10) = 50$
$\therefore x = 50$

Clave B

35.



Se sabe:

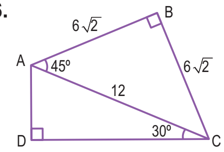
$$\frac{AD}{AC} = \frac{5}{4}$$

$$\frac{AD}{8\sqrt{2}} = \frac{5}{4}$$

$\therefore AD = 10\sqrt{2}$ m

Clave C

36.



$AC = (6\sqrt{2})\sqrt{2} = 12$

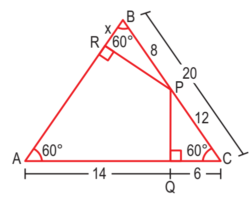
$AD = \frac{12}{2}$

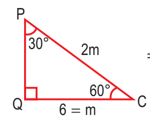
$\therefore AD = 6$

Clave A

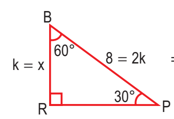
Resolución de problemas

37. Piden: RB





$\Rightarrow PC = 12$

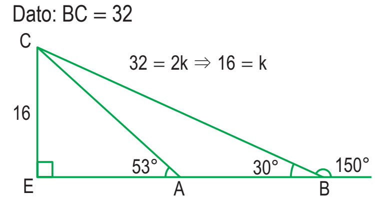


$\Rightarrow k = 4$

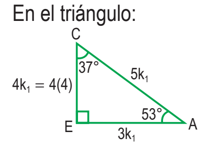
$\therefore RB = x = 4$

Clave B

38. Piden: perímetro del ⊿AEC
Dato: BC = 32



En el triángulo:

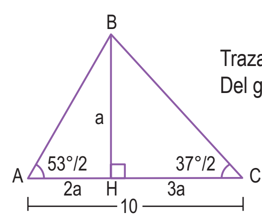


$\Rightarrow k_1 = 4$

$\Rightarrow 2p = 4k_1 + 5k_1 + 3k_1$
$\therefore 2p = 12k_1 = 12(4) = 48$

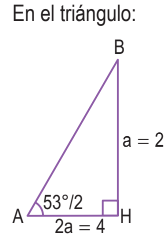
Clave B

39. Piden: AB



Trazamos la altura BH.
Del gráfico: $5a = 10$
$a = 2$

En el triángulo:

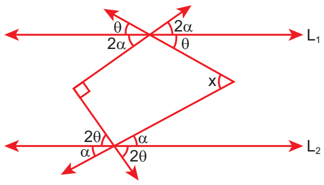


Por Pitágoras:
$4^2 + 2^2 = (AB)^2$
$16 + 4 = (AB)^2$
$\sqrt{20} = \sqrt{(AB)^2}$
$\therefore AB = 2\sqrt{5}$

Clave B

MARATÓN MATEMÁTICA (página 33)

1. Trasladamos los ángulos en la región interior de las rectas paralelas por ángulos opuestos por el vértice luego:

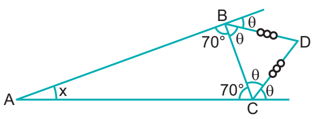


$2\alpha + 2\theta = 90°$
$\alpha + \theta = 45°$ ... (I)

De la misma manera:
$\theta + \alpha = x$
pero de (I):
$\theta + \alpha = 45° = x \Rightarrow x = 45°$

Clave C

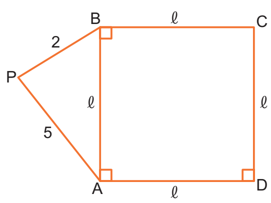
2. Si $BD = DC \Rightarrow$ el $\triangle BDC$ es isósceles.
Por lo tanto, $m\angle DBC = m\angle DCB = \theta$
pero $\overline{BD}$ y $\overline{CD}$ son bisectrices.



$\therefore\ 2\theta + m\angle ABC = 180$
$2\theta + 70° = 180°$
$\theta = 55°$
Luego, $m\angle ACB + 2(55°) = 180°$
$\Rightarrow\ m\angle ACB = 70°$
$\therefore$ En el $\triangle ABC$: $x + 70° + 70° = 180$
$\Rightarrow\ x = 40°$

Clave D

3. Por el postulado de existencia de triángulos:
En el $\triangle PBA$.



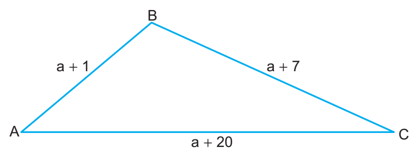
Tenemos que:
$5 - 2 < \ell < 5 + 2$
$3 < \ell < 7$

Por lo tanto, "ℓ" puede tomar los valores enteros: 4; 5; 6 $\Rightarrow\ \ell_{máx.} = 6$ ... (I)

Luego, el perímetro dle cuadrado ABCD será
$2p = 4\ell$
$\therefore\ 2p_{máx.} = 4(\ell_{máx});$
de (I): $2p_{máx.} = 4 \times 6$
$2p_{máx.} = 24$

Clave A

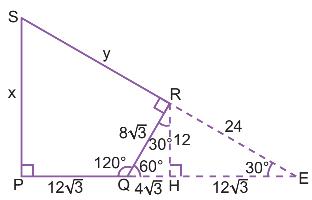
4. En el $\triangle ABC$ aplicamos el postulado de la existencia de triángulos



$BC - AB < AC < BC + AB$
Reemplazando:
$\Rightarrow (a + 7) - (a + 1) < a + 20 < (a + 1) + (a + 7)$
$a + 7 - a - 1 < a + 20 < a + 1 + a + 7$
$\underbrace{6 < a + 20 < 2a + 8}$
$20 - 8 < 2a - a \Rightarrow 12 < a$
$\therefore\ a_{mín.} = 13$

Clave C

5. Prolongamos $\overline{PQ}$ y $\overline{SR}$ de tal manera que sus prolongaciones se intersecan en E; luego vemos que el ⊿ERQ es notable de 30° y 60°; ya que $m\angle RQE = 60°$ y $m\angle REQ = 30°$



Si $QR = 8\sqrt{3}$
$\Rightarrow\ QE = 2(8\sqrt{3})$ y $ER = 8\sqrt{3}\,(\sqrt{3})$
$QE = 16\sqrt{3}$ y $ER = 24$

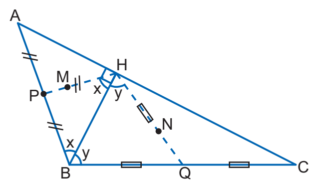
Luego, también vemos que el ⊿SPE también es notable de 30° y 60°, por lo tanto,

si $PE = 12\sqrt{3} + 16\sqrt{3} = 28\sqrt{3}$

$\Rightarrow\ SP = 28 = x$ y $SE = 2(28)$
pero $SE = y + 24 = 2(28)$
$\Rightarrow y = 32$
Nos piden $SP + SR = x + y$
$SP + SR = 28 + 32$
$\Rightarrow\ SP + SR = 60$

Clave B

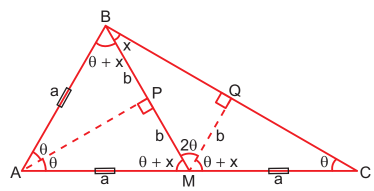
6. Prolongamos $\overline{HM}$ y $\overline{HN}$ hasta que intersecan al lado $\overline{AB}$ y al lado $\overline{BC}$ en los puntos P y Q respectivamente.



$\Rightarrow\ \overline{HP}$ y $\overline{HQ}$ son medianas relativas a la hipotenusa de los triángulos rectángulos AHB y BHC; por lo tanto, se cumple:
$HP = AP = PB$ y $HQ = CQ = QB$
$\Rightarrow$ Los triángulos PHB y QHB son isósceles, por lo tanto:
$m\angle PHB = m\angle PBH = x\ \wedge$
$m\angle QHB = m\angle QBH = y$
Nos piden $x + y$ pero
$m\angle ABC = 110° = x + y$
$\Rightarrow\ x + y = 110°$

Clave C

7. En el $\triangle ABM$ trazamos la mediatriz AP, luego sabemos que $AB = AM = a$ y $BP = PM = b$, luego trazamos la altura MQ perpendicular a $\overline{BC}$. Como BM es mediana:



$\Rightarrow\ AM = MC = a$
pero: $2\theta + x = 90°$
$\Rightarrow\ QM = b$

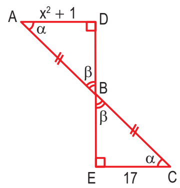
Luego, en el ⊿BQM tenemos que $BM = 2b$ y $QM = b$
$\therefore$ ⊿BQM es notable de 30° y 60° $\Rightarrow\ x = 30°$

Clave C

Unidad 2

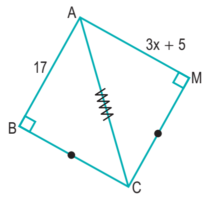
CONGRUENCIA DE TRIÁNGULOS

APLICAMOS LO APRENDIDO (página 34) Unidad 2

1. 

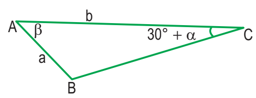
Completando ángulos, se tiene:
$\triangle ADB \cong \triangle CEB$ (ALA)
$\Rightarrow AD = EC$
$x^2 + 1 = 17$
$x^2 = 16$
$\therefore x = 4$

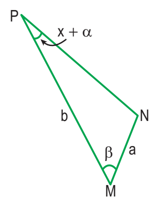
Clave D

2. 

De la figura:
$\triangle ABC \cong \triangle AMC$ (LLL)
$\Rightarrow AB = AM$
$17 = 3x + 5$
$12 = 3x$
$\therefore x = 4$

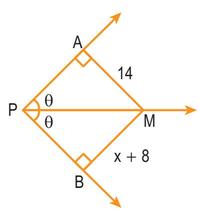
Clave B

3. 



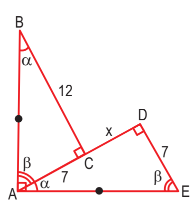
De la figura:
$\triangle ABC \cong \triangle MNP$ (LAL)
$\Rightarrow 30° + \alpha = x + \alpha$
$\therefore x = 30°$

Clave A

4. 

Como $\overline{PM}$ es bisectriz del $\angle APB$, por el teorema de la bisectriz:
$MA = MB$
$14 = x + 8$
$\therefore x = 6$

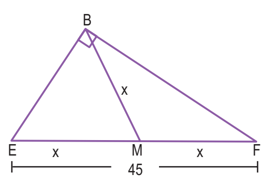
Clave A

5. 

Como AB = AE, al completar ángulos tenemos:
$\triangle ACB \cong \triangle EDA$ (ALA)
$\Rightarrow AC = DE = 7$

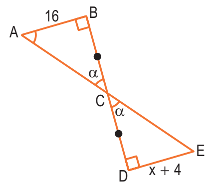
También:
$BC = AD$
$12 = 7 + x$
$\therefore x = 5$

Clave B

6. 

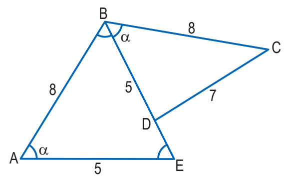
Por teorema de la mediana relativa a la hipotenusa:
$BM = EM = MF$
De la figura:
$x + x = 45$
$2x = 45$
$\therefore x = 22,5$

Clave C

7. 

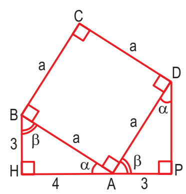
El $\triangle ABC \cong \triangle EDC$ (ALA)
$\Rightarrow x + 4 = 16$
$\therefore x = 12$

Clave C

8. 

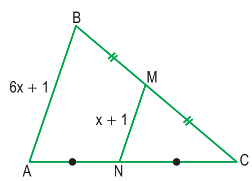
El $\Delta EAB \cong \Delta DBC$ (LAL)
Luego:
$BD + DE = 7$
$5 + DE = 7$
$DE = 2$

Clave D

9. 

Completando ángulos, tenemos:
$\triangle BHA \cong \triangle APD$ (ALA)
$\Rightarrow BH = AP = 3$
Por dato:
$PH = 7$
$\Rightarrow AH = 4$
Luego: $\triangle$ BHA notable 37° y 53°
$\Rightarrow a = 5$
Nos piden:
Perímetro ABCD = 4a
= 4(5)
= 20

Clave A

10. 

En el ΔABC se observa:
$\overline{MN}$ es base media

$$\Rightarrow MN = \frac{AB}{2}$$

$$x + 1 = \frac{6x + 1}{2}$$

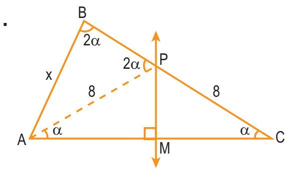
$$2x + 2 = 6x + 1$$

$$\Rightarrow x = \frac{1}{4}$$

Luego nos piden:

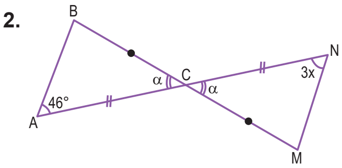
$$(x+1)^2 = \left(\frac{1}{4} + 1\right)^2 = \left(\frac{5}{4}\right)^2 = \frac{25}{16}$$

Clave B

11. 

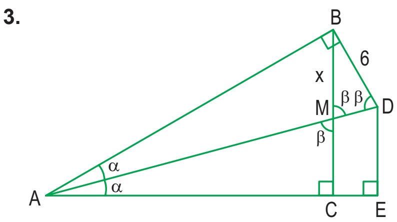
Como $\overleftrightarrow{PM}$ es mediatriz:
$\Rightarrow AM = MC$
También al trazar $\overline{AP}$:
$AP = PC = 8$
y $m\angle PAC = m\angle PCA = \alpha$
Del gráfico, se observa:
El ΔPAB es isósceles:
$\Rightarrow AB = AP$
$\therefore x = 8$

Clave C

12. 

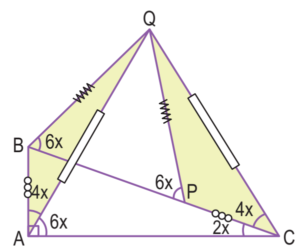
Completando ángulos, tenemos:
$\Delta BCA \cong \Delta MCN$ (LAL)
$\Rightarrow m\angle BAC = m\angle MNC$
$46° = 3x$
$\therefore x = 15,33°$

Clave C

13. 

Sea el complemento de α el ángulo $\beta \Rightarrow \alpha + \beta = 90°$
$\Rightarrow m\angle ADB = \beta$ y $m\angle AMC = \beta$
$\Rightarrow$ El ΔMDB es isósceles
$\therefore BM = BD = 6$
$\Rightarrow x = 6$

Clave A

14. 

Como el BQP es isósceles $\Rightarrow \overline{BQ} \cong \overline{QP}$ y también el AQC es isósceles $\Rightarrow \overline{AQ} \cong \overline{QC}$
$\Rightarrow$ el $\Delta BQA \cong \Delta PQC$, caso LLL (dato: PC = AB)
$m\angle QAB \; m\angle QCP = 4x$
$m\angle BAC = 90° \Rightarrow 4x + 6x = 90° \Rightarrow x = 9°$

Clave D

PRACTIQUEMOS

Nivel 1 (página 36) Unidad 2

Comunicación matemática

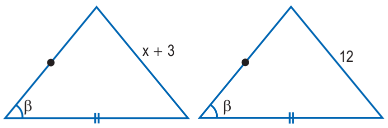
1. VFV

2.

3.

Razonamiento y demostración

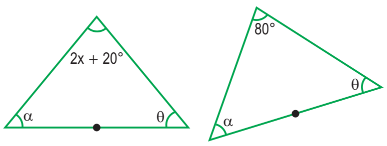
4.



x + 3 = 12
x = 9

Clave E

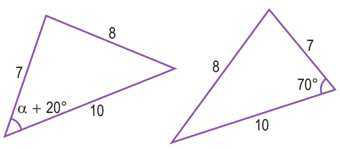
5.



2x + 20° = 80°
2x = 60°
x = 30°

Clave E

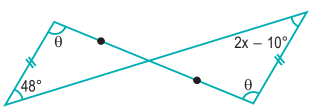
6.



α + 20° = 70°
α = 50°

Clave D

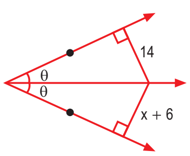
7.



2x – 10° = 48°
2x = 58°
x = 29°

Clave C

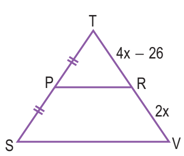
8.



x + 6 = 14
x = 8

Clave E

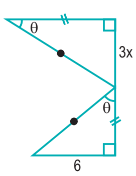
9.



4x – 26 = 2x
2x = 26
x = 13

Clave C

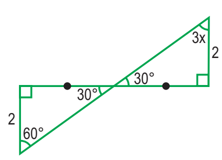
10.



3x = 6
$x = \frac{6}{3}$
x = 2

Clave E

11.

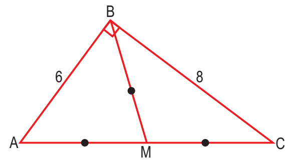


3x = 60°
x = 20°

Clave D

Resolución de problemas

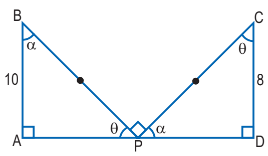
12.



BM es mediana relativa a la hipotenusa, entonces:
AM = MC = BM
AC = 10 (Por pitágoras)
⇒ 2AM = 10
AM = 5
∴ BM = 5

Clave E

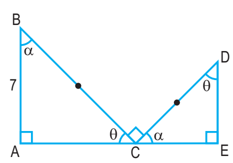
13.



◺BAP ≅ ◺PDC (ALA)
Entonces:
AP = 8
PD = 10
∴ AD = 18

Clave A

14.



◺BAC ≅ ◺CED
(ALA)
∴ CE = 7

Clave B

15. Notamos que:
◺AEB ≅ ◺BDC
Entonces: DC = BE
↓ ↓
x = BD + DE
x = 4 + 2 = 6

Clave D

Nivel 2 (página 37) Unidad 2

Comunicación matemática

16.

17.

18.

Razonamiento y demostración

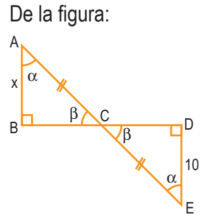
19. De los triángulos congruentes (LLL):
θ = 95°

Clave B

20. De los triángulos congruentes (LLL):
x = 70°

Clave B

21. De la figura:



◺ABC ≅ ◺EDC
Entonces:
AB = DE ⇒ x = 10

Clave C

22. De ambos triángulos congruentes:
∴ α = 23°

Clave E

23. Los triángulos son congruentes (LAL).
∴ α = 20°

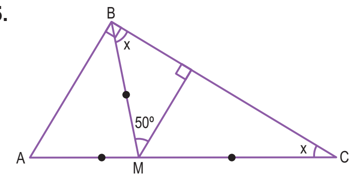
Clave E

24. Por propiedad:
$x = \frac{8+8}{2}$
∴ x = 8

Clave C

Resolución de problemas

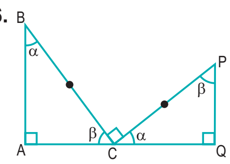
25.



$\overline{BM}$ es mediana relativa a la hipotenusa, entonces:
AM = MC = BM
Luego:
x + 50° = 90°
∴ x = 40°

Clave B

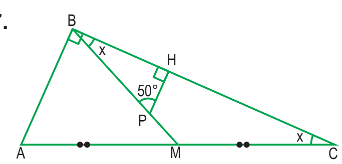
26.



Dato: BA + PQ = 10
◺ BAC ≅ ◺ CQP
(AL A)
Entonces:
BA = CQ
PQ = AC
BA + PQ = AQ
∴ AQ = 10

Clave C

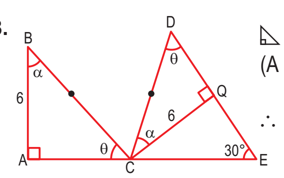
27.



Dato: $\overline{BM}$ es mediana:
En el ◺PHB: 50° + x = 90°
∴ x = 40°

Clave C

28.



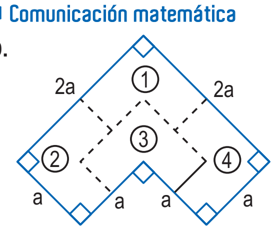
◺ BAC ≅ ◺ CQD
(A L A)
CE = 6(2)
∴ CE = 12

Clave E

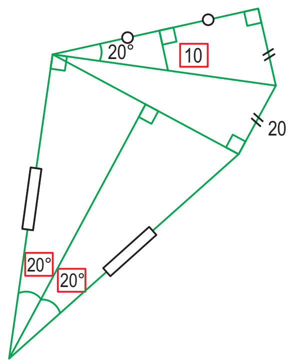
Nivel 3 (página 38) Unidad 2

Comunicación matemática

29.



30.



31.

Razonamiento y demostración

32. De los triángulos congruentes (LLL):

$\alpha = 15°$ **Clave C**

33. Por propiedad:

$\therefore x = 18°$ **Clave D**

34. Propiedad de la base media:

$\therefore\ x = 8$ **Clave C**

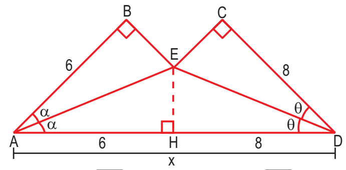
35. Por congruencia (L A L):

$\therefore x = 12$ **Clave D**

36. Por congruencia de triángulos (A L A):

$8 + 3 = x \Rightarrow x = 11$ **Clave E**

37.

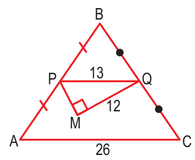


Trazamos $\overline{EH}$, perpendicular a $\overline{AD}$.
Por el teorema de la bisectriz:
$AB = AH = 6\ \wedge\ CD = DH = 8$
Del gráfico:
$x = 6 + 8 = 14$
$\therefore x = 14$

Clave C

Resolución de problemas

38.



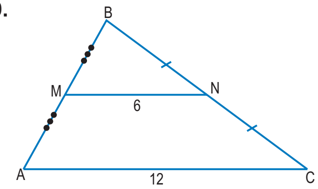
$\overline{PQ}$: base media

$PQ = \dfrac{AC}{2} = \dfrac{26}{2} = 13$

Por el teorema de Pitágoras:
$PM^2 = 13^2 - 12^2$
$PM^2 = 25$
$\therefore\ PM = 5$

Clave C

39.

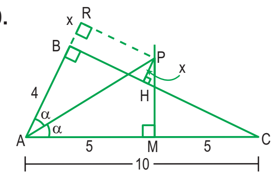


Por dato: $AB + BC = 20$
Por el teorema de los puntos medios:
$AC = 12$
Piden:
$2p_{\triangle ABC} = AB + BC + AC = 20 + 12 = 32$

$\therefore 2p_{\triangle ABC} = 32$

Clave C

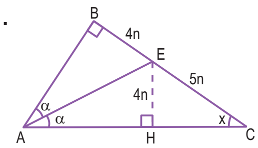
40.



Por el teorema de la bisectriz:
$4 + x = 5$
$\therefore\ x = 1$

Clave A

41.



El $\triangle$EHC, notable de 37° y 53°.
$\therefore x = 53°$

Clave E

POLÍGONOS

APLICAMOS LO APRENDIDO (página 40) Unidad 2

1. $m\angle i = \dfrac{180°(n-2)}{n}$

$\Rightarrow\ \dfrac{180°(n-2)}{n} = 150°$

$18n - 36 = 15n$
$3n = 36$
$n = 12$

Por lo tanto, el polígono es un dodecágono.

Clave D

2. $n = 12$
$Sm\angle i = 180°(n-2)$
$Sm\angle i = 180°(12-2)$
$Sm\angle i = 1800°$

Clave D

3. $n = 6$

$m\angle i = \dfrac{180°(n-2)}{n}$

$m\angle i = \dfrac{180°(6-2)}{6}$

$m\angle i = 120°$ **Clave B**

4. $Sm\angle i\ = 120°$; hallamos n:

$120° = \dfrac{180°(n-2)}{n}$

$12n = 18n - 36$
$6n = 36$
$n = 6$

$D_T = \dfrac{n(n-3)}{2} = \dfrac{6(3)}{2} = 9$

Clave B

5. $D_T = 119$

$\dfrac{n(n-3)}{2} = 119$

$n(n-3) = 238$
$n = 17$

Clave C

6. $n = 18°$

$m\angle i = \dfrac{180°(n-2)}{n}$

$m\angle i = \dfrac{180°(18-2)}{18}$

$m\angle i = 160°$ **Clave C**

7. Piden: nombre del polígono.
Dato: $Sm\angle i = 1080°$
Sabemos:
$180°(n-2) = 1080°$
$(n-2) = 6$
$\Rightarrow\ n = 8$
Por lo tanto, el polígono es un octógono.

Clave A

8. Piden: n.° lados
Dato: $m\angle e = 72°$
Sabemos:

$\dfrac{360°}{n} = 72°$

$\dfrac{360}{72} = n \Rightarrow n = 5$

$\therefore$ n.° lados es 5.

Clave E

9. Piden: D_T
Dato: cuadrilátero $\Rightarrow n = 4$
Sabemos:

$D_T = \dfrac{n(n-3)}{2}$

$\therefore D_T = \dfrac{4(4-3)}{2} = 2$

Clave E

10. Piden: D_T
Dato: octógono ⇒ n = 8
Sabemos:
$$D_T = \frac{n(n-3)}{2}$$
$$\therefore D_T = \frac{8(5)}{2} = 20$$
Clave A

11. Piden: D_T
Dato: $\sum m\angle i = 1980°$
$180°(n-2) = 1980°$
$n - 2 = 11$
$\Rightarrow n = 13$
$$D_T = \frac{n(n-3)}{2}$$
$$\therefore D_T = \frac{13(10)}{2} = 65$$
Clave B

12. Piden: nombre del polígono.
Datos: $m\angle e = \frac{1}{5}(90°) = 18°$
Entonces: $18° = \frac{360°}{n}$
$n(18°) = 360°$
$n = 20$
Por lo tanto, es un icoságono.
Clave C

13. Piden: nombre del polígono.
Dato: $D_T = 35$
Sabemos:
$$35 = \frac{n(n-3)}{2}$$
$70 = n(n-3)$
$10 . 7 = n(n-3)$
$\Rightarrow n = 10$
Por lo tanto, es un decágono.
Clave B

14. Piden: número de diagonales.
Dato: hexágono ⇒ n = 6
Sabemos:
$$D_T = \frac{n(n-3)}{2}$$
$$D_T = \frac{6(3)}{2} \qquad \therefore D_T = 9$$
Clave A

PRACTIQUEMOS
Nivel 1 (página 42) Unidad 2

Comunicación matemática

1.
2.
3.

Razonamiento y demostración

4. $Sm\angle i = 180°(6-2)$
$Sm\angle i = 180°(4)$
$Sm\angle i = 720°$
Clave B

5. $Sm\angle i = 180°(45-2)$
$Sm\angle i = 180°(43)$
$Sm\angle i = 7740°$
Clave A

6. En todo polígono la suma de ángulos exteriores es 360°.
Clave C

7. $D_T = \frac{10}{2}(10-3)$
$D_T = 5(7) \quad \Rightarrow D_T = 35$
Clave D

8. $Sm\angle i = 180°(28-2)$
$Sm\angle i = 180°(26)$
$Sm\angle i = 4680°$
Clave D

Resolución de problemas

9. Piden: nombre del polígono.
Dato: $S_i = 1260°$
Entonces:
$180°(n-2) = 1260°$
$n - 2 = 7$
$\Rightarrow n = 9$
Por lo tanto
Es un nonágono.
Clave E

10. Piden: número de diagonales.
Dato: tiene 10 ángulos internos ⇒ n = 10
Sabemos:
$$D = \frac{n(n-3)}{2}$$
$$D = \frac{10(7)}{2} = 35$$
Clave B

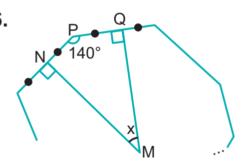
11. Piden: D_T
Datos: $\angle i = 135°$
Sabemos:
$$\angle i = \frac{180°(n-2)}{n}$$
$$\Rightarrow 135° = \frac{180°(n-2)}{n}$$
$$3 = \frac{4(n-2)}{n}$$
$3n = 4n - 8$
$n = 8$
También:
$$D_T = \frac{n(n-3)}{2}$$
$$D_T = \frac{8(8-3)}{2} \qquad \therefore D = 20$$
Clave A

12. Sea el número de lados del polígono: n
Por dato: ND = 6(n)
$$\Rightarrow \frac{n(n-3)}{2} = 6n$$
$n - 3 = 12 \qquad \Rightarrow n = 15$
Por lo tanto, el polígono es un pentadecágono.
Clave E

13. Sea n el número de lados del polígono regular.
Por dato:
$m\angle e = 40°$
$$\frac{360°}{n} = 40° \Rightarrow n = \frac{360°}{40°}$$
$n = 9$
Piden: n.° de diagonales (ND)
$$ND = \frac{n(n-3)}{2} = \frac{9(9-3)}{2} \qquad \therefore ND = 27$$
Clave D

14. Piden: $\angle c$
Dato: n.° vértices = D
n.° vértices = n (lados)
Entonces:
$$n = \frac{n(n-3)}{2}$$
$2n = n(n-3)$
$n = 5$
$$\therefore \angle c = \frac{360°}{n} = \frac{360°}{5} = 72°$$
Clave E

15. Piden: n.° vértices
Dato: D + n = 105
$$\frac{n(n-3)}{2} + n = 105$$
$n(n-3) + 2n = 210$
$n(n-1) = 210$
$n(n-1) = 15(14)$
$\Rightarrow n = 15$
∴ n.° vértices es 15.
Clave C

16.


n = 9 lados
$$\angle i = \frac{180°(n-2)}{n} = \frac{180°(9-2)}{9} = 140°$$
En el polígono MNPQ de 4 lados:
$x + 90° + 90° + 140° = 180°(4-2)$
$x + 320° = 360°$
$\therefore x = 40°$
Clave B

Nivel 2 (página 43) Unidad 2

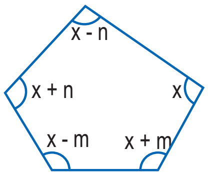
Comunicación matemática

17.
18.
19.

Razonamiento y demostración

20. $D_T = \frac{30}{2}(30-3)$
$D_T = 15(27)$
$D_T = 405$
Clave A

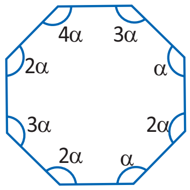
21.



$5x = 180°(5 - 2)$
$5x = 540°$
$x = 108°$

Clave E

22.



$18\alpha = 180°(8 - 2)$
$18\alpha = 1080°$
$\alpha = 60°$

Clave A

23. Piden: número de diagonales.
Dato: icoságono tiene 20 lados.
Sabemos:

$$D_T = \frac{n(n-3)}{2}$$

$$D_T = \frac{20(17)}{2} \qquad \therefore\ D = 170$$

Clave D

24. Piden: $m\angle i$
Dato: dodecágono equiángulo tiene 12 lados.
Sabemos:

$$m\angle i = \frac{180°(n-2)}{n}$$

$$m\angle i = \frac{180°(10)}{12}$$

$$m\angle i = 150°$$

Clave D

Resolución de problemas

25. Sea n el número de lados del polígono
Del enunciado:

$$\frac{n(n-3)}{2} + n = 2n$$

$$\frac{n(n-3)}{2} = n$$

$$n - 3 = 2 \qquad \therefore\ n = 5$$

Clave B

26. $m\angle e = \dfrac{360°}{n} = 40° \Rightarrow n = 9$

$$D_T = \frac{9(9-3)}{2} = 27$$

Clave D

27. $\dfrac{n(n-3)}{2} + 2n = 6$

$n(n-3) + 4n = 12$
$n^2 + n - 12 = 0$
$\left.\begin{matrix} n & +4 \\ n & -3 \end{matrix}\right\} \Rightarrow n = 3$

Clave D

28. D_T + n.° vértices = 2(n.° de lados)

$$\frac{n}{2}(n-3) + n = 2n$$

$$\frac{n}{2}(n-3) = n$$

$$n = 5$$

Clave B

29. n.° ángulos externos: n
$n_1 = 2n$
$n_2 = n$
Entonces, del enunciado:
$180°(2n - 2) = 3 \cdot 180°(n - 2)$
$2n - 2 = 3n - 6$
$4 = n$
$Sm\angle i = 180°(8 - 2) = 1080°$

Clave C

30. $n_1 + n_2 = 12$ .... (α)

$$\frac{m\angle e_1}{m\angle e_2} = \frac{2}{1}$$

$$\frac{\frac{360°}{n_1}}{\frac{360°}{n_2}} = \frac{2}{1} \Rightarrow \frac{n_2}{n_1} = 2 \text{ ....}(\beta)$$

(β) en (α): $3n_1 = 12$

$n_1 = 4 \ \wedge\ n_2 = 8$

$$x = \frac{\frac{180°(4-2)}{4}}{\frac{180°(8-2)}{8}} = \frac{2}{3}$$

Clave A

Nivel 3 (página 43) Unidad 2

Comunicación matemática

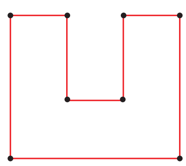
31.

32.

33.

Razonamiento y demostración

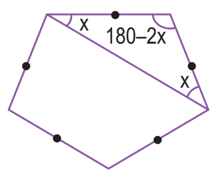
34. Piden: Si



n = 8
Sabemos:
$Si = 180°(n - 2)$
$Si = 180°(8 - 2)$
$Si = 1080°$

Clave A

35.



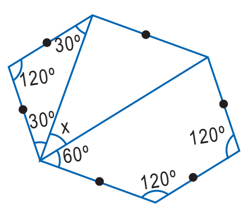
Como es un polígono regular:

$$180° - 2x = \frac{180(5-2)}{5}$$

$2x = 72°$
$\therefore\ x = 36°$

Clave A

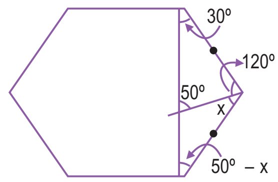
36.



Como el polígono es regular:
$x + 90° = 120°$
$\therefore x = 30°$

Clave A

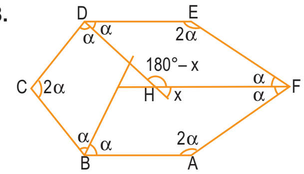
37.



Del gráfico:
$50 - x° = 30°$
$\therefore\ x = 20°$

Clave D

38.



Por suma de ángulos internos:
$6(2\alpha) = 720°$
$\alpha = 60°$
En el cuadrilátero DEFH:
$180° - x + 60° + 60° + 120° = 360°$
$420° - x = 360°$
$\therefore\ x = 60°$

Clave E

Resolución de problemas

39. Piden: n.° de vértices
Dato:
$Sm\angle i + Sm\angle e = 1980°$
Sabemos:
$180°(n - 2) + 360° = 1980°$
$180(n - 2) = 1620$
$(n - 2) = 9$
$n = 11$
Como n.° vértices = n
Por lo tanto:
n.° vértices es 11.

Clave E

40. Piden: número de diagonales
Dato: $\dfrac{m\angle i}{m\angle e} = \dfrac{7}{2}$

Sabemos:

$$\frac{\frac{180°(n-2)}{n}}{\frac{360°}{n}} = \frac{7}{2}$$

$$\frac{180(n-2)}{360} = \frac{7}{2}$$

$360(n - 2) = 7 \cdot 360$
$n - 2 = 7$
$\Rightarrow\ n = 9$

Por lo tanto:

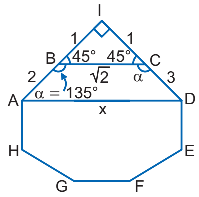
$$D_T = \frac{n(n-3)}{2} = \frac{9 \cdot 6}{2} = 27$$

Clave A

41. Sea n el número de lados del polígono.
Dato: $Sm\angle i_1 = 180°(n - 2)$
$Sm\angle i_2 = 180°(n + 4 - 2)$
$\Rightarrow Sm\angle i_2 = 2Sm\angle i_1$
$180°(n + 2) = 2 \cdot 180°(n - 2)$
$n + 2 = 2n - 4$
$\therefore\ n = 6$

Clave B

42. Piden: AD

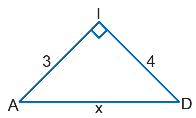


n = 8

$$\alpha = \frac{180°(n-2)}{n}$$

$$= \frac{180°(8-2)}{8}$$

$\alpha = 135°$



$x^2 = 3^2 + 4^2$

$x^2 = 25 \Rightarrow x = 5$

$\therefore$ AD = 5

Clave C

43. Piden: $\angle c$

Dato: lado = 6 $\Rightarrow$ 2p = 6n

2p = 6n = D

Entonces:

$$6n = \frac{n(n-3)}{2}$$

12 = n − 3

15 = n

$$\therefore \angle c = \frac{360°}{n} = \frac{360°}{15} = 24°$$

Clave D

44. Piden: n

Dato: $Dm + D_T = 35$

Sabemos:

$$\frac{n(n-1)}{2} + \frac{n(n-3)}{2} = 35$$

n(n − 1) + n(n − 3) = 70

n(2n − 4) = 70

n(2n − 4) = 7 (10)

n = 7

Por lo tanto:

Es un heptágono.

Clave C

CUADRILÁTEROS

APLICAMOS LO APRENDIDO (página 45) Unidad 2

1. 6x + 7x + x + 30° + 50° = 360°

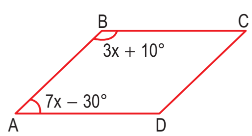
14x + 80° = 360°

14x = 280°

x = 20°

Clave B

2.



7x − 30° + 3x + 10° = 180°

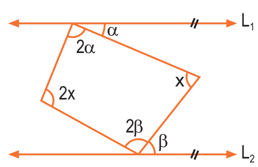
10x = 180° + 30° − 10°

10x = 200°

x = 20°

Clave B

3.



Por propiedad:

x = α + β

Además:

2x + 2α + x + 2β = 360°

3x + 2(α + β) = 360°

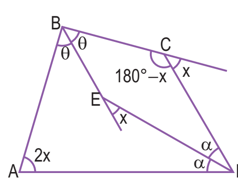
3x + 2(x) = 360°

5x = 360°

$\therefore$ x = 72°

Clave E

4.



En el cuadrilátero BCDE se cumple:

x + x = θ + α ⇒ θ + α = 2x

En el cuadrilátero ABCD:

2x + 2θ + 180° − x + 2α = 360°

x + 2(θ + α) = 180°

x + 2(2x) = 180°

5x = 180°

$\therefore$ x = 36°

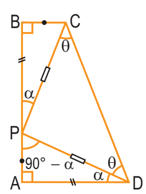
Clave B

5. Por propiedad:

$$\therefore z = \frac{110° + 70°}{2} = 90°$$

Clave D

6.



Del gráfico:

⊿CBP ≅ ⊿PAD (LAL)

⇒ m∠BPC = m∠ADP = α

Se deduce: m∠CPD = 90°

Además: CP = PD

⇒ m∠PCD = m∠PDC = θ

En el ⊿CPD:

θ + θ = 90° ⇒ 2θ = 90°

$\therefore$ θ = 45°

Clave D

7. Dato:

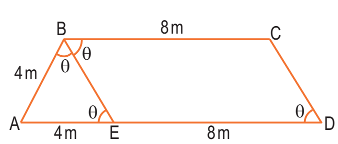
m∠A − m∠C = 22°

Utilizamos propiedad:

$$x = \frac{m\angle A - m\angle C}{2} = 11°$$

Clave B

8.



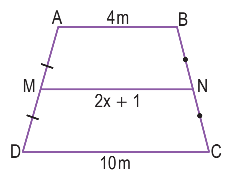
Trazamos $\overline{BE}$ // $\overline{CD}$

El △ABE resulta isósceles.

Luego: AD = 8 + 4 = 12m

Clave A

9.



$\overline{MN}$: mediana del trapecio

$$\frac{10+4}{2} = 2x + 1$$

7 = 2x + 1

2x = 6

$\therefore$ x = 3m

Clave C

10. 8y + 140° = 180°

y = 5

5x + 12° + 3x + 8° = 180°

8x = 180° − 12° − 8°

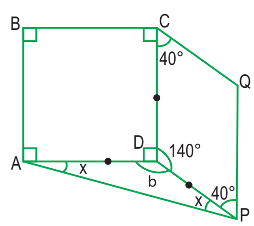
8x = 160°

x = 20°

$$\Rightarrow \frac{y}{x} = \frac{5°}{20°} = \frac{1}{4}$$

Clave B

11.



90° + 140° + β = 360°

β = 130°

En el △ADP:

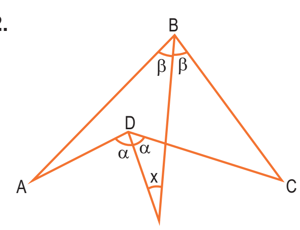
2x + β = 180°

2x = 50°

$\therefore$ x = 25°

Clave A

12.



Propiedad: $x = \dfrac{m\angle C - m\angle A}{2}$

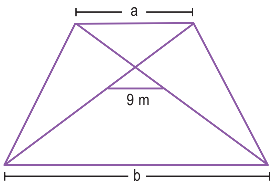
Por dato: $m\angle C - m\angle A = 32°$

$\Rightarrow x = \dfrac{32°}{2}$

$\therefore\ x = 16°$

Clave B

13.



$a + b = 30\,m$ ... (1)

Además sabemos:

$\dfrac{b-a}{2} = 9\,m \Rightarrow b - a = 18\,m$ ... (2)

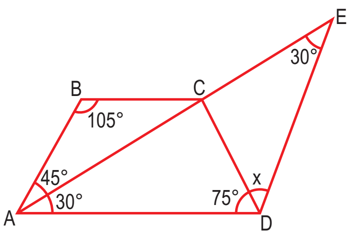
De (1) y (2) tenemos:

$2b = 48$

$b = 24\,m$

Clave D

14.



$75° + 30° + 30° + x = 180°$

$x = 45°$

Clave A

PRACTIQUEMOS

Nivel 1 (página 47) Unidad 2

Comunicación matemática

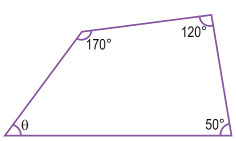
1.

2.

3.

Razonamiento y demostración

4.



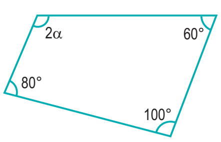
$\theta + 170° + 170° = 360°$

$\theta = 360° - 340°$

$\theta = 20°$

Clave B

5.



$2\alpha + 100° + 140° = 360°$

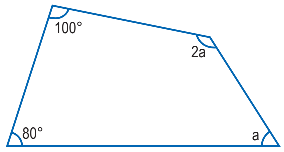
$2\alpha = 360° - 240°$

$2\alpha = 120°$

$\alpha = 60°$

Clave D

6.



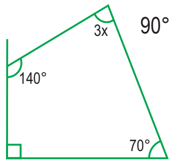
$100° + 2a + 80° + a = 360°$

$3a = 180°$

$a = 60°$

Clave B

7.



$90° + 210° + 3x = 360°$

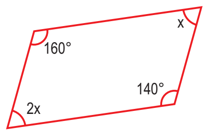
$3x = 360° - 300°$

$3x = 60°$

$x = 20°$

Clave D

8.



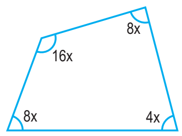
$3x + 300° = 360°$

$3x = 60°$

$x = 20°$

Clave C

9.



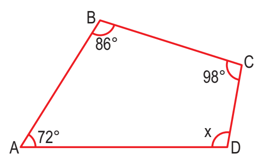
$36x = 360°$

$x = 10°$

Clave E

Resolución de problemas

10.



En el cuadrilátero ABCD:

$72° + 86° + 98° + x = 360°$

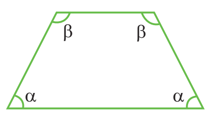
$256° + x = 360°$

$\therefore\ x = 104°$

Clave C

11. Por propiedad:

Sea el trapecio isósceles:



$\alpha + \beta = 180°$

$\downarrow$

$60° \Rightarrow 60° + \beta = 180°$

$\therefore\ \beta = 120°$

Clave A

12. Sean las bases: a, b

$a + b + \dfrac{a+b}{2} = 45$

$\Rightarrow \dfrac{2(a+b)+(a+b)}{2} = 45$

$3(a+b) = 90 \Rightarrow a + b = 30$

Por lo tanto:

La mediana es: $\dfrac{a+b}{2} = \dfrac{30}{2} = 15\,m$

Clave D

13. Sea el lado menor: a

$\Rightarrow 5 + a + 5 + a = 16$

$2a = 6$

$\therefore\ a = 3\,cm$

Clave B

14. Sean los ángulos:

2a; 3a; 5a; 8a

$\Rightarrow 2a + 3a + 5a + 8a = 360°$

$18a = 360°$

$\Rightarrow a = 20°$

Por tanto:

El ángulo menor es: $2a = 40°$

Clave B

15. Sea M la mediana del trapecio:

$\Rightarrow M = \dfrac{18+10}{2} = \dfrac{28}{2} = 14\,m$

Clave D

Nivel 2 (página 48) Unidad 2

Comunicación matemática

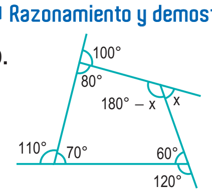
16.

17.

18.

Razonamiento y demostración

19.



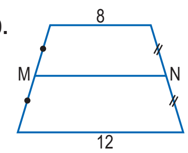
$80° + 70° + 60° + 180° - x = 360°$

$390° - x = 360°$

$\Rightarrow\ x = 30°$

Clave C

20.



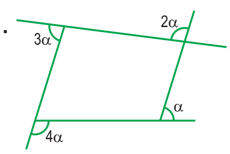
$MN = \dfrac{12+8}{2}$

$MN = \dfrac{20}{2}$

$MN = 10$

Clave E

21.

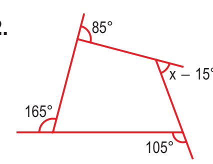


$10\alpha = 360°$

$\alpha = 36°$

Clave E

22.



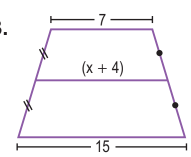
$355° + x - 15° = 360°$

$340° + x = 360°$

$x = 20°$

Clave D

23.



$x + 4 = \dfrac{7+15}{2}$

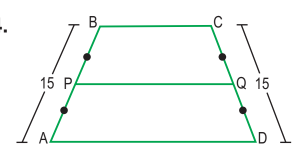
$x + 4 = 11$

$x = 7$

Clave C

Resolución de problemas

24.



Dato:

perímetro = 50

$BC + AD + 30 = 50$

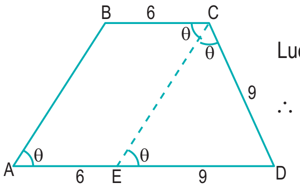
$BC + AD = 20$

PQ: mediana del trapecio

$$PQ = \frac{BC + AD}{2}$$

$\therefore\ PQ = 10$

Clave B

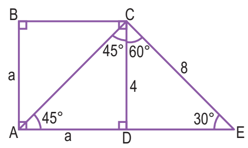
25. 

Luego:

$AD = 6 + 9$

$\therefore\ AD = 15\,m$

Clave A

26. 

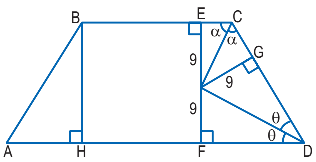
$CD = \frac{8}{2} = 4$

Perímetro del cuadrado ABCD = 4CD

Perímetro del cuadrado ABCD = 4(4)

$\therefore$ Perímetro del cuadrado ABCD = 16 m

Clave C

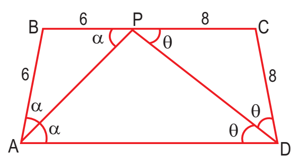
27. 

Del gráfico:

$BH = EF$

$BH = 9 + 9 \quad \therefore\ BH = 18$

Clave B

28. 

Del gráfico:

$CP = CD = 8$

$BP = AB = 6 \quad \therefore\ BC = 14$

Clave B

Nivel 3 (página 49) Unidad 2

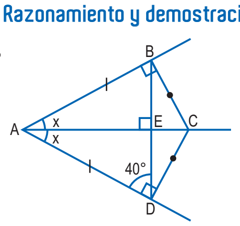
Comunicación matemática

29.

30.

31.

Razonamiento y demostración

32. 

Por el teorema de la bisectriz:

$BC = CD \ \wedge\ AB = AD$

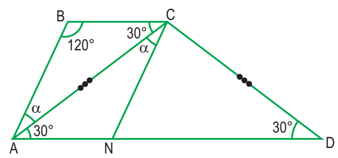
Entonces el cuadrilátero ABCD es un trapezoide simétrico, luego:

$m\angle AEB = 90°$

En el ◺AED:

$x + 40° = 90° \qquad \therefore\ x = 50°$

Clave E

33. 

Por dato ABCD es un trapecio ($\overline{BC} \,//\, \overline{AD}$)

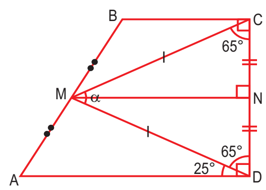
$\Rightarrow m\angle NAC = m\angle ACB = 30°$

En el △ABC:

$\alpha + 120° + 30° = 180°$

$\alpha + 150° = 180° \qquad \therefore\ \alpha = 30°$

Clave A

34. 

Tenemos la base media MN:

$\Rightarrow\ CN = ND$

Luego $\Delta MNC \cong \Delta MND$

$\Rightarrow\ \angle MCN = 65°$

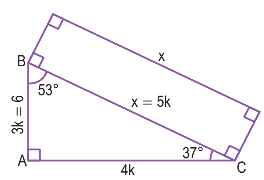
En el ΔCMD:

$\alpha + 65° + 65° = 180°$

$\alpha + 130° = 180°$

$\therefore\ \alpha = 50°$

Clave C

35. 

Del gráfico:

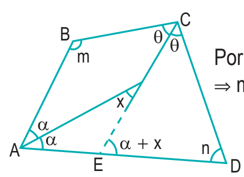
$3k = 6 \Rightarrow k = 2$

Luego:

$x = 5k = 5(2)$

$\therefore\ x = 10$

Clave B

36. 

Por dato: $m = n + 62°$

$\Rightarrow m - n = 62° \quad ...\ (1)$

En el cuadrilátero ABCD:

$m + n + 2(\alpha + \theta) = 360°$

$\Rightarrow \alpha + \theta = 180° - \left(\frac{m+n}{2}\right) \quad ...\ (2)$

En el △CED:

$\alpha + x + \theta + n = 180°$

$\Rightarrow \alpha + \theta = 180° - (x + n) \quad ...\ (3)$

De (2) y (3):

$\frac{m+n}{2} = x + n$

$m + n = 2x + 2n$

$\Rightarrow x = \frac{m - n}{2} \quad ...\ (4)$

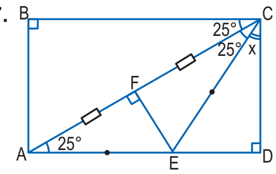
Reemplazando (1) en (4):

$\Rightarrow x = \frac{m-n}{2} = \frac{62°}{2} = 31°$

$\therefore\ x = 31°$

Clave E

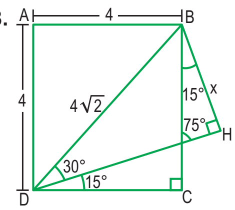
Resolución de problemas

37. 

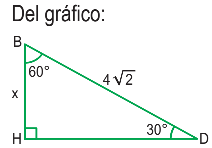
Del gráfico, el ΔAEC isósceles, entonces el $\angle ACE = 25°$

Por lo tanto $x = 40°$

Clave D

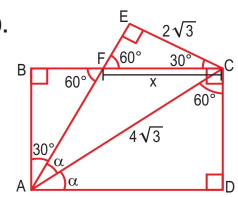
38. 

Del gráfico:



$\Rightarrow x = 2\sqrt{2}$

Clave D

39. 

Del gráfico:

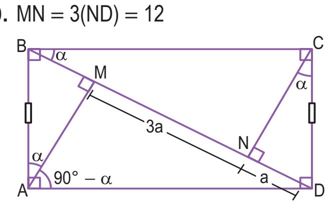
$\alpha = 30°$

Del ◺FEC:

$x = 4$

Clave A

40. $MN = 3(ND) = 12$



Aplicamos congruencia:

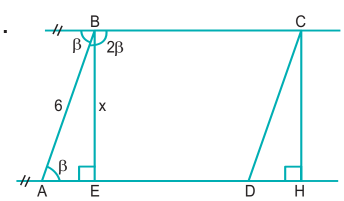
◺AMB ≅ ◺CND $\Rightarrow$ BM = a

Por dato: $MN = 12 = 3a$

$\Rightarrow a = 4$

Luego: $BD = 5a = 5(4) = 20\,cm$

Clave B

41. 

Por ser paralelogramo se cumple:

$3\beta = 180° \Rightarrow \beta = 60°$

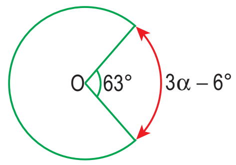
Del △AEB, notable de 30° y 60°:

$x = 3\sqrt{3}\ m$

Clave B

APLICAMOS LO APRENDIDO (página 51) Unidad 2

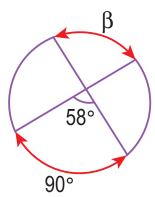
1.



$3\alpha - 6° = 63°$
$3\alpha = 69°$
$\alpha = 23°$

Clave C

2.

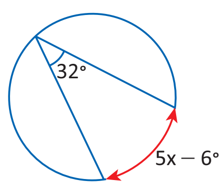


$58° = \dfrac{90° + \beta}{2}$
$116° = 90° + \beta$
$\beta = 26°$

Piden:
$2\beta - 10° = 2(26°) - 10° = 42°$

Clave A

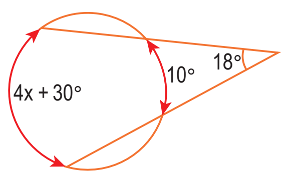
3.



$32° = \dfrac{5x - 6°}{2}$
$64° = 5x - 6°$
$70° = 5x$
$x = 14°$

Clave E

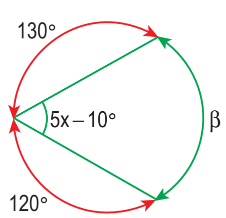
4.



$18° = \dfrac{4x + 30° - 10°}{2}$
$36° = 4x + 20°$
$4x = 16°$
$x = 4°$

Clave D

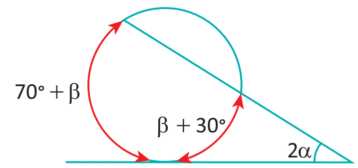
5.



$\beta = 360° - (120° + 130°)$
$\beta = 110°$
$5x - 10° = \dfrac{\beta}{2}$
$5x - 10° = \dfrac{110°}{2}$
$5x - 10° = 55°$
$5x = 65°$
$x = 13°$

Clave A

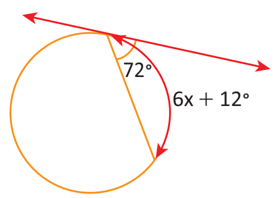
6.



$2\alpha = \dfrac{70° + \beta - \beta - 30°}{2}$
$2\alpha = \dfrac{40°}{2}$
$2\alpha = 20° \Rightarrow \alpha = 10°$

Clave E

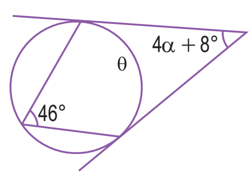
7.



$72° = \dfrac{6x + 12°}{2}$
$144° = 6x + 12° \Rightarrow x = 22°$

Clave A

8.

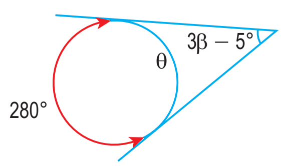


Hallamos primero θ:
$46° = \dfrac{\theta}{2} \Rightarrow \theta = 92°$

Luego:
$4\alpha + 8° = \dfrac{268° - 92°}{2}$
$8\alpha + 16° = 176°$
$8\alpha = 160° \Rightarrow \alpha = 20°$

Clave B

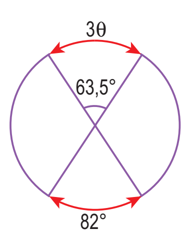
9.



El valor de θ es: $360° - 280° = 80°$
Entonces:
$3\beta - 5° = \dfrac{280° - 80°}{2}$
$3\beta - 5° = 100°$
$3\beta = 105°$
$\beta = 35°$

Clave B

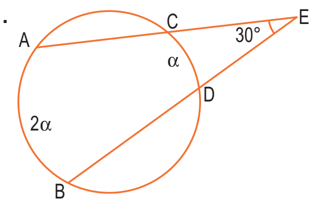
10.



Del gráfico, por propiedad, del ángulo interior:
$63{,}5° = \dfrac{3\theta + 82°}{2}$
$127° = 3\theta + 82°$
$45° = 3\theta$
$\therefore\ \theta = 15°$

Clave D

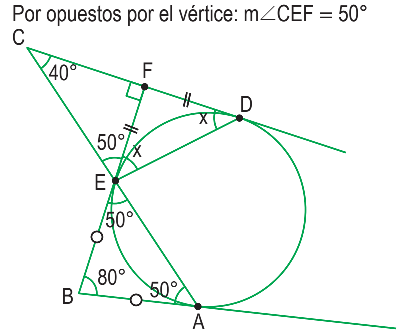
11.



Del dato:
$m\overset{\frown}{AB} = 2m\overset{\frown}{CD}$
Sea $\overset{\frown}{CD} = \alpha \Rightarrow m\overset{\frown}{AB} = 2\alpha$
Por ángulo exterior:
$30° = \dfrac{2\alpha - \alpha}{2} \Rightarrow \alpha = 60°$
Piden:
$m\overset{\frown}{CD} = \alpha \quad \therefore\ m\overset{\frown}{CD} = 60°$

Clave B

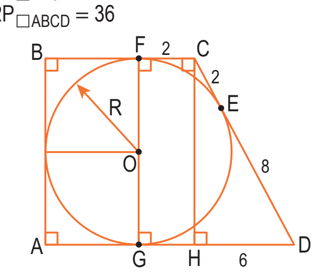
12. Trazamos la cuerda $\overline{ED}$ y prolongamos $\overline{BE}$ hasta que interseca a $\overline{CD}$ en el punto F. Luego por propiedad sabemos que el △ABE es isósceles (AB = BE).
Entonces: $80° + 2m\angle BEA = 180°$
$m\angle BEA = 50°$
Por opuestos por el vértice: $m\angle CEF = 50°$



Del gráfico:
$m\angle CFE = 90°$
Luego por propiedad:
△EFD es isósceles (FE = FD)
$\Rightarrow m\angle FDE = m\angle FED = x$
$\therefore\ x + x = 90° \Rightarrow x = 45°$

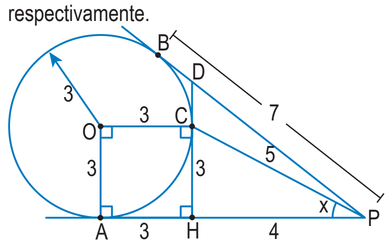
Clave C

13. Trazamos $\overline{CH}$ perpendicular a $\overline{AD}$, luego por propiedad FC = CE = 2 y ED = GD = 8, pero como □FCHG es un rectángulo, entonces:
FC = GH = 2, pero GD = GH + HD
$8 = 2 + HD \Rightarrow HD = 6$
Luego, el △CHD es pitagórico:
CD = 10, HD = 6
$\Rightarrow$ CH = 8; pero CH = 2R $\Rightarrow$ R = 4,
Luego en el □ABCD: AB = 8, BC = 4 + 2 = 6, CD = 10, AD = 8 + 4 = 12
$\Rightarrow 2P_{\square ABCD} = 8 + 6 + 10 + 12$
$\Rightarrow 2P_{\square ABCD} = 36$



Clave B

14. Trazamos $\overline{OA}$ y $\overline{OC}$ perpendicular a $\overline{AP}$ y a $\overline{DH}$ respectivamente.



Luego AOCH es un cuadrado de lado 3
$\Rightarrow$ AH = 3
Por propiedad BP = AP = 7
$\Rightarrow$ AP = AH + HP
$7 = 3 + HP \Rightarrow HP = 4$;
Luego el $\triangle$CHP es triángulo notable de 37° y 53°
$\therefore$ x = 37°

Clave C

PRACTIQUEMOS
Nivel 1 (página 53) Unidad 2

Comunicación matemática

1.

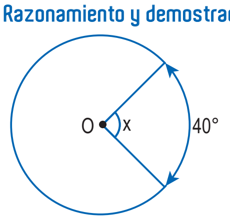
2.

3.

4.

Razonamiento y demostración

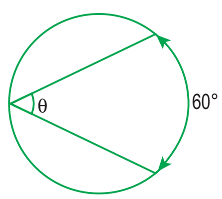
5.



$x = 40°$

Clave C

6.

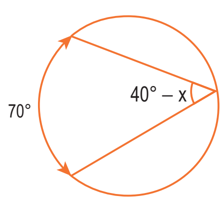


$\theta = \dfrac{60°}{2}$

$\theta = 30°$

Clave D

7.



$40° - x = \dfrac{70°}{2}$

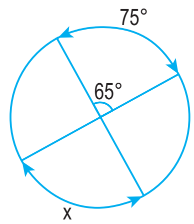
$40° - x = 35°$

$40° - 35° = x$

$\Rightarrow x = 5°$

Clave D

8.



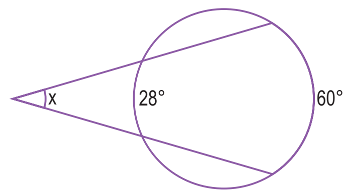
$65° = \dfrac{x + 75°}{2}$

$130° = x + 75°$

$\Rightarrow x = 55°$

Clave E

9.



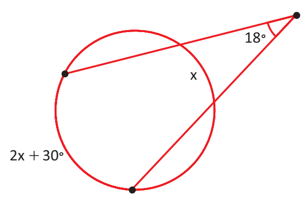
$x = \dfrac{60° - 28°}{2}$

$x = \dfrac{32°}{2}$

$x = 16°$

Clave E

10.



$18° = \dfrac{2x + 30° - x}{2}$

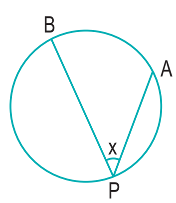
$36° = x + 30°$

$\Rightarrow 6° = x$

Clave A

Resolución de problemas

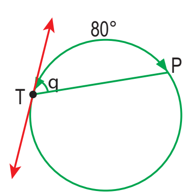
11. Piden: x



Dato: $m\overset{\frown}{AB} = 70°$
Por propiedad:
$m\overset{\frown}{AB} = 2x \Rightarrow 2x = 70°$
$\therefore x = 35°$

Clave D

12. Piden: θ

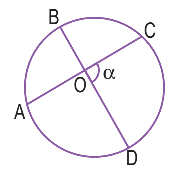


Dato: $m\overset{\frown}{PT} = 80°$
Por propiedad:
$m\overset{\frown}{PT} = 2\theta = 80°$
$\therefore \theta = 40°$

Clave D

13. Piden: α

Dato: $m\overset{\frown}{AB} = 60°$ y $m\overset{\frown}{CD} = 80°$



Por propiedad:

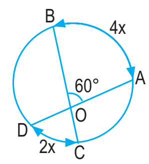
$\alpha = \dfrac{m\overset{\frown}{AB} + m\overset{\frown}{CD}}{2}$

$\alpha = \dfrac{60° + 80°}{2} \quad \therefore \alpha = 70°$

Clave E

14. Piden: $m\overset{\frown}{CD}$

Datos: $m\overset{\frown}{AB} = 4x$ y $m\overset{\frown}{CD} = 2x$



Por propiedad:

$m\angle DOC = \dfrac{m\overset{\frown}{AB} + m\overset{\frown}{CD}}{2}$

$\Rightarrow 60° = \dfrac{4x + 2x}{2}$

$60° = 3x$

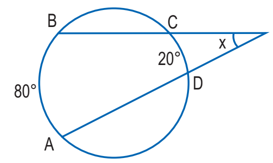
$x = 20°$

$\therefore m\overset{\frown}{CD} = 2(20°) = 40°$

Clave E

15. Piden: x

Datos: $m\overset{\frown}{AB} = 80°$ y $m\overset{\frown}{CD} = 20°$



Del gráfico:

$x = \dfrac{m\overset{\frown}{AB} - m\overset{\frown}{CD}}{2}$

$x = \dfrac{80° - 20°}{2} = 30°$

Clave A

Nivel 2 (página 54) Unidad 2

Comunicación matemática

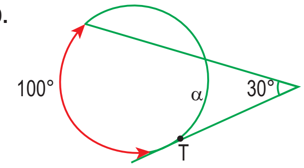
16.

17.

18.

Razonamiento y demostración

19.



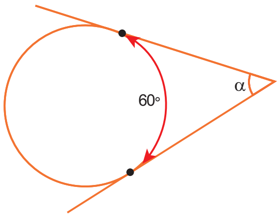
$30° = \dfrac{100° - \alpha}{2}$

$60° = 100° - \alpha$

$\alpha = 40°$

Clave A

20.

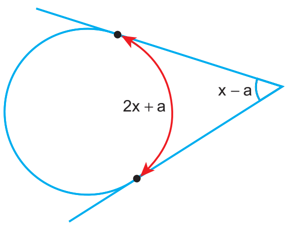


$\alpha + 60° = 180°$

$\alpha = 120°$

Clave B

21.



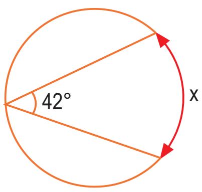
$x - a + 2x + a = 180°$

$3x = 180°$

$x = 60°$

Clave A

22.

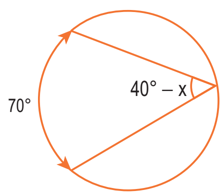


$\dfrac{x}{2} = 42°$

$x = 84°$

Clave E

23.



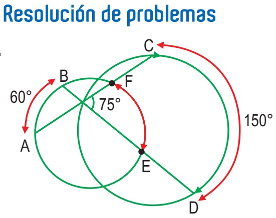
$x + 13° = \dfrac{70°}{2}$

$x = 22°$

Clave C

Resolución de problemas

24.



Por ángulo interior:

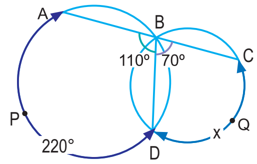
$\dfrac{60° + m\overset{\frown}{FE}}{2} = 75°$

$m\overset{\frown}{FE} = 150° - 60°$

$\therefore m\overset{\frown}{FE} = 90°$

Clave D

25.



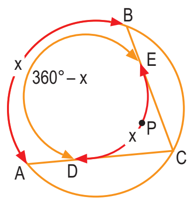
Por ángulo inscrito:

$m\angle ABD = 110° \Rightarrow m\angle DBC = 70°$

$\therefore x = m\overset{\frown}{CQD} = 140°$

Clave C

26.



Del gráfico:

$m\angle BCA = \dfrac{(360° - x) - x}{2} = \dfrac{x}{2}$

$180° - x = \dfrac{x}{2}$

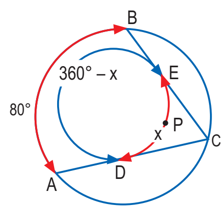
$360° - 2x = x$

$360° = 3x$

$\therefore x = 120°$

Clave D

27.



$m\angle BCA = \dfrac{80°}{2} = \dfrac{(360° - x) - x}{2}$

$2x = 280°$

$\therefore x = m\overset{\frown}{EPD} = 140°$

Clave C

Nivel 3 (página 55) Unidad 2

Comunicación matemática

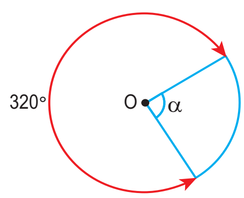
28.

29.

30.

Razonamiento y demostración

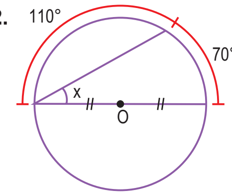
31.



$\alpha = 40°$

Clave E

32.

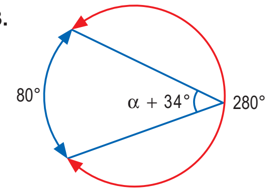


$x = \dfrac{70°}{2}$

$x = 35°$

Clave B

33.



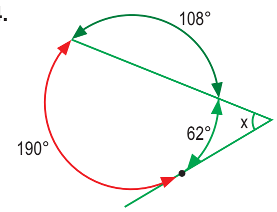
$\alpha + 34° = \dfrac{80°}{2}$

$\alpha + 34° = 40°$

$\alpha = 6°$

Clave D

34.



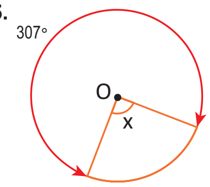
$x = \dfrac{190° - 62°}{2}$

$x = \dfrac{128°}{2}$

$x = 64°$

Clave E

35.



$x = 53°$

Clave E

Resolución de problemas

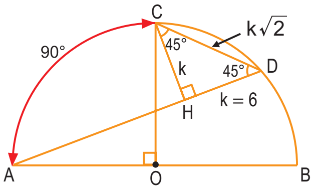
36. $5x + 45° = 180°$

$5x = 135°$

$x = 27°$

Clave D

37.



Del gráfico: $m\widehat{AC} = 90°$

Por ángulo inscrito:

$m\angle CDA = \dfrac{m\widehat{AC}}{2} = \dfrac{90°}{2} = 45°$

$\Rightarrow m\angle CDA = 45°$

Entonces el ⊿CHD es notable de 45°.

Luego: k = 6

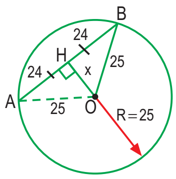
Piden:

$CD = k\sqrt{2} = 6\sqrt{2}$

$\therefore\ CD = 6\sqrt{2}$

Clave E

38.



El $\triangle$TAOB es isósceles, entonces:

AH = HB = 24

En el ⊿OHB por el teorema de Pitágoras:

$252 = 242 + x^2$

$\Rightarrow x^2 = 49$

$\therefore\ x = 7$ m

Clave D

39. $2x + 150° = 180°$

$2x = 30°$

$x = 15°$

Clave D

40. $m\widehat{AC} + m\widehat{CB} = 180°$

$\Rightarrow\ m\widehat{CB} = 40°$

$2x = 20°$

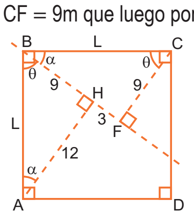
$x = 10°$

Clave D

MARATÓN MATEMÁTICA (página 57)

41. De los datos del enunciado tenemos:

AH = 12m y CF = 9m que luego por inspección:



$m\angle BAH = m\angle CBF = \alpha$ y

$m\angle ABH = m\angle BCF = \theta$ y también AB = BC = L

$\therefore$ ⊿ABH ≅ ⊿BCF

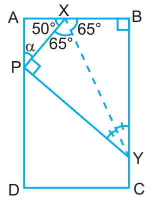
$\Rightarrow$ BH = CF = 9 m y AH = BF = 12 m

Finalmente en el ⊿ABH: $L^2 = 9^2 + 12^2$

$\therefore\ L = 15$ m

Clave E

42. Trazamos $\overline{XP}$ y $\overline{PY}$; luego el $\triangle$XPY se origina al doblar la hoja por $\overline{XY}$ de modo que el punto B se superponga al punto P.



$\therefore\ \overline{XP} \cong XB$ y $\overline{PY} \cong \overline{BY}$

entonces ⊿XPY ≅ ⊿ XBY (caso LLL)

Sabemos que

$m\angle XPy = m\angle XBY = 90°$ y

$m\angle BXY = m\angle PXY = 65°$

Del dato tenemos:

$m\angle AXY + m\angle BXY = 180°$

$115° + m\angle BXY = 180°$

$\Rightarrow\ m\angle BXY = 65°$

Finalmente en el PAX: $\alpha + 50° = 90 \Rightarrow \alpha = 40°$

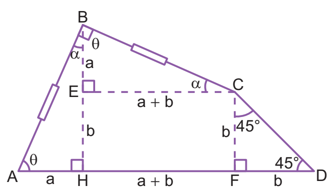
Clave B

43. Trazamos $\overline{BH}$ perpendicular a $\overline{AD}$ y también $\overline{CE}$ perpendicular a $\overline{BH}$; luego del dato:

$m\angle ABC = 90° \Rightarrow m\angle ABH = \alpha$ y $m\angle HBC = \theta$

$\therefore\ \alpha + \theta = 90 \Rightarrow m\angle BCE = \alpha$ y $m\angle BAH = \theta$

También $\overline{AB} \cong \overline{BC}$



Por lo tanto: ⊿AHB ≅ ⊿BEC (caso ALA)

$\Rightarrow$ AH = BE = a ... (I)

Luego trazamos $\overline{CF}$ perpendicular a $\overline{AD}$

$\Rightarrow$ ⊿CFD es isósceles

$\therefore$ CF = FD = b $\Rightarrow$ como ECFH es un rectángulo

$\Rightarrow$ EH = CF = b

De (I) como ⊿AHB ≅ ⊿BEC

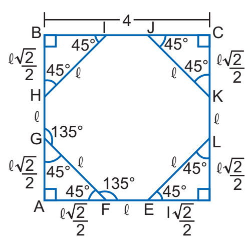
$\Rightarrow$ BH = EC = HF = a + b; del dato: AD = L

Pero $2(a + b) = AD \Rightarrow a + b = L/2$

$\therefore\ BH = L/2$

Clave B

44. Cuando inscribimos un octógono regular dentro de un cuadrado debemos asegurarnos de que todos los vértices del octógono se encuentren dentro de los lados de dicho cuadrado.



Luego el octógono regular EFGHIJKL determina cuatro triángulos congruentes y notables de 45°.

$\Rightarrow$ Si $GF = \ell \Rightarrow GA = AF = ED = LD = \ell\dfrac{\sqrt{2}}{2}$

Finalmente $AD = \ell\dfrac{\sqrt{2}}{2} + \ell + \ell\dfrac{\sqrt{2}}{2} = 4$

$\therefore \ell = 4(\sqrt{2} - 1)$

Clave B

45. Decimos que a es la medida de un ángulo interno y b la medida de un ángulo externo de un polígono equiángulo; por lo tanto:

$a = \dfrac{180°(n-2)}{n}$ y $b = \dfrac{360°}{n}$

dividimos ambos expresiones:

$$\frac{a}{b} = \frac{\dfrac{180°(n-2)}{n}}{\dfrac{360°}{n}}$$

$$\frac{a}{b} = \frac{(n-2)}{2}$$

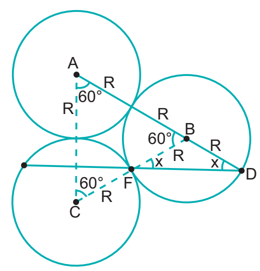
$2\left(\dfrac{a}{b}\right) = n - 2$

$n = 2\left(\dfrac{a}{b} + 1\right)$

Clave B

46. Trazamos los segmentos AC y CB, los cuales pasan por los puntos de tangencia de las circunferencias al igual que AB, luego como las circunferencias son congruentes poseen el mismo radio R; por inspección vemos que el $\triangle$ABC es equilátero, pues:

AB = BC = AC = 2R



Entonces se cumple:

$\therefore\ m\angle CAB = m\angle ABC = m\angle BCA = 60°$

Luego el triángulo FDB es isósceles:

FB = BD = R $\Rightarrow m\angle BFD = m\angle BDF = x$

Por ángulo externo: $2x = 60°$

$\Rightarrow\ x = 30°$

Clave D

APLICAMOS LO APRENDIDO (página 60) Unidad 3

1. Sea: FE = x

Por el teorema de la bisectriz exterior:

$\dfrac{LV}{LE} = \dfrac{VF}{FE}$; reemplazando:

$$\frac{13}{11+x} = \frac{7}{x} \Rightarrow 13x = 77 + 7x$$
$$6x = 77$$
$$x = 12{,}83$$

Clave C

2. Se debe cumplir que:

$$\frac{x}{x+4} = \frac{4}{x-2}$$

Resolviendo:

$x^2 - 2x = 4x + 16$

$x^2 - 6x = 16$

$x(x-6) = 8(2)$

$\therefore\ x = 8$

Clave C

3. De la figura:

$$\frac{1{,}8}{6} = \frac{1{,}2}{4} = \frac{z}{5}$$

Resolviendo:

$$z = \frac{5 \times 1{,}8}{6} \Rightarrow z = 1{,}5$$

Clave B

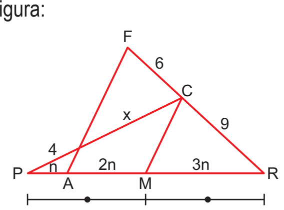
4. Por propiedad:

$$\frac{9}{x} = \frac{6}{1{,}5}$$
$$\Rightarrow x = \frac{9 \times 1{,}5}{6}$$

$\therefore\ x = 2{,}25$

Clave A

5. De la figura:

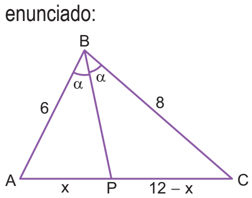


Entonces:

$$\frac{4}{n} = \frac{x}{2n} \Rightarrow x = 8$$

Clave C

6. Según el enunciado:

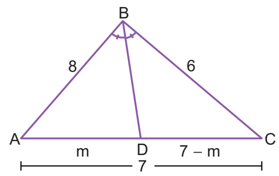


Entonces:

$$\frac{6}{x} = \frac{8}{12-x} \Rightarrow x = \frac{36}{7}$$

Clave E

7. Del enunciado:



Del teorema de la bisectriz interior:

$$\frac{AB}{AD} = \frac{BC}{DC} \Rightarrow \frac{8}{m} = \frac{6}{7-m} \Rightarrow m = 4$$

Luego:

$AD = 4 \wedge DC = 3$

$\therefore\ AD - DC = 1$

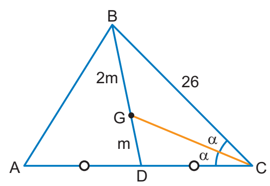
Clave C

8. Por el teorema de Thales:

$$\frac{AB}{BC} = \frac{DE}{EF} \Rightarrow \frac{3}{a} = \frac{b}{7} \quad \therefore\ ab = 21$$

Clave D

9.



G es baricentro y $\overline{BD}$ es mediana

$$\frac{BG}{GD} = 2$$

AD = DC

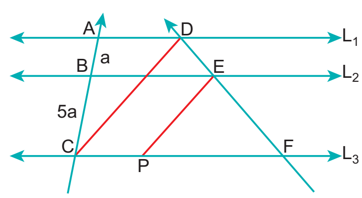
Teorema de la bisectriz interior: (ΔBGC)

$$\frac{BC}{BG} = \frac{CD}{GD} \Rightarrow \frac{BC}{CD} = \frac{BG}{GD} \Rightarrow \frac{26}{CD} = 2$$

$\Rightarrow\ CD = AD = 13 \quad \therefore\ AC = 26$

Clave D

10.



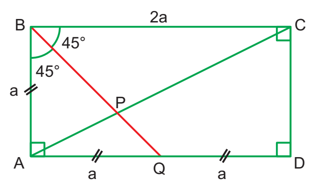
$$\frac{DE}{EF} = \frac{AB}{BC} \qquad \text{... (I)}$$
$$\frac{CP}{PF} = \frac{DE}{EF} \qquad \text{... (II)}$$

De (I) y (II)

$$\frac{CP}{PF} = \frac{AB}{BC} \quad \therefore\ \frac{CP}{PF} = \frac{1}{5}$$

Clave B

11. ⊿BAQ isósceles:



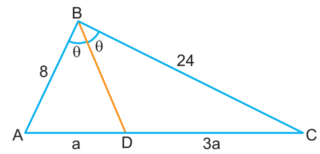
$\Rightarrow\ AB = AQ = QD = a$

BP es bisectriz, por teorema de la bisectriz interior:

$$\frac{AB}{AP} = \frac{BC}{PC}$$
$$\therefore\ \frac{AP}{PC} = \frac{AB}{BC} = \frac{1}{2}$$

Clave D

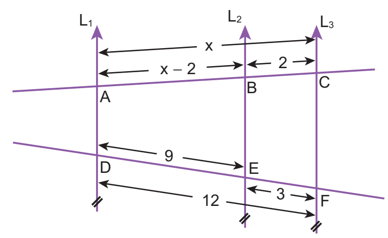
12. Por teorema de la bisectriz interior:



$$\frac{AB}{AD} = \frac{BC}{DC} \Rightarrow \frac{8}{AD} = \frac{24}{DC}$$
$$\frac{DC}{AD} = 3 \Rightarrow DC = 3a \wedge AD = a$$
$$\therefore\ \frac{DC}{AD} = \frac{3a}{4a} = \frac{3}{4}$$

Clave B

13. Del gráfico:



$AB = x - 2 \wedge EF = 3$

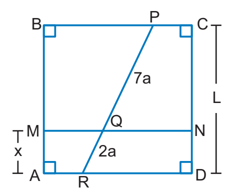
Por teorema de Thales:

$$\frac{AB}{BC} = \frac{9}{3} \Rightarrow \frac{x-2}{2} = 3$$

$\therefore\ x = 8$

Clave C

14. De los datos:



$$\frac{PQ}{QR} = \frac{7}{2} \Rightarrow PQ = 7a \wedge QR = 2a$$

Por teorema de Thales

$$\frac{x}{L} = \frac{2a}{9a}$$
$$\therefore\ x = \frac{2}{9}L$$

Clave E

Practiquemos

Nivel 1 (página 62) Unidad 3

Comunicación matemática

1.

2.

3.

Razonamiento y demostración

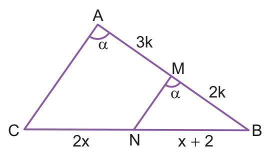
4. $\dfrac{x+1}{x+2} = \dfrac{4}{5}$

$5x + 5 = 4x + 8$

$x = 3$

Clave C

5.



$\dfrac{3k}{2k} = \dfrac{2x}{x+2} \Rightarrow 4x = 3x + 6$

$\therefore\ x = 6$

Clave D

6. $\dfrac{8}{x} = \dfrac{24}{27} \Rightarrow x = 9$

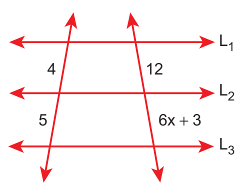
Piden: $x + 3$

$\Rightarrow x + 3 = 9 + 3$

$x + 3 = 12$

Clave C

7.



$\dfrac{4}{5} = \dfrac{12}{6x+3}$

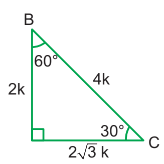
$6x + 3 = 15$

$6x = 12 \Rightarrow x = 2$

Clave B

8. Por ⊿ notable:

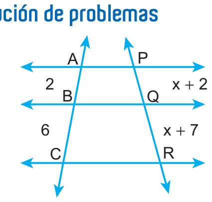
$\therefore x = 30°$



Clave B

Resolución de problemas

9.



$\dfrac{2}{6} = \dfrac{x+2}{x+7} \Rightarrow \dfrac{1}{3} = \dfrac{x+2}{x+7}$

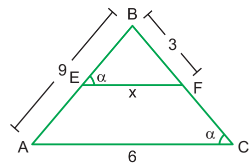
$x + 7 = 3x + 6$

$2x = 1$

$\therefore\ x = \dfrac{1}{2} = 0{,}5$

Clave B

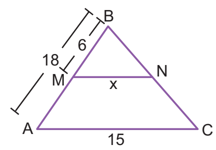
10.



$\dfrac{3}{x} = \dfrac{9}{6} \Rightarrow 2 = x$

Clave A

11.

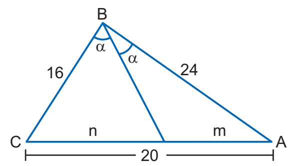


$\dfrac{18}{6} = \dfrac{15}{x} \Rightarrow 3x = 15$

$\therefore x = 5$

Clave A

12.



$m + n = 20 \ldots \text{(I)}$

Por proporcionalidad:

$\dfrac{24}{m} = \dfrac{16}{n} \Rightarrow \dfrac{3}{m} = \dfrac{2}{n} \Rightarrow m = \dfrac{3n}{2}$

Reemplazando en (I):

$\dfrac{3n}{2} + n = 20 \Rightarrow n = 8\ \wedge\ m = 12$

Clave B

Nivel 2 (página 63) Unidad 3

Comunicación matemática

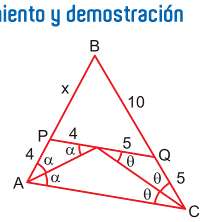
13.

14.

15.

Razonamiento y demostración

16.

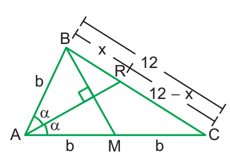


$\dfrac{x}{4} = \dfrac{10}{5} \Rightarrow \dfrac{x}{4} = 2$

$\therefore x = 8$

Clave D

17.



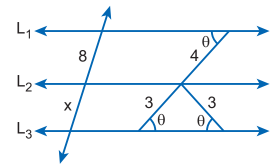
$\dfrac{x}{b} = \dfrac{12 - x}{2b}$

$2x = 12 - x$

$\therefore x = 4$

Clave A

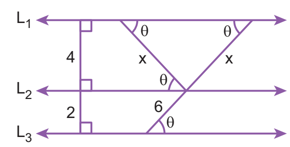
18.



$\dfrac{8}{x} = \dfrac{4}{3} \Rightarrow x = 6$

Clave D

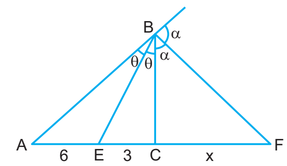
19.



$\dfrac{4}{2} = \dfrac{x}{6} \Rightarrow x = 12$

Clave C

20.



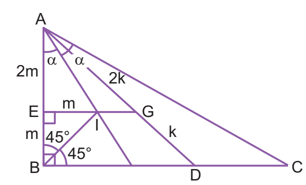
$\dfrac{6}{3} = \dfrac{9+x}{x} \Rightarrow 2x = 9 + x$

$\therefore x = 9$

Clave B

Resolución de problemas

21.



Como $\overline{IG} \parallel \overline{BC}$:

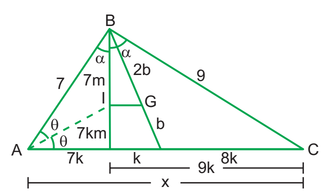
$\dfrac{AE}{EB} = \dfrac{AG}{GD} = \dfrac{2}{1}$ (propiedad baricentro)

En el $\triangle AEI$, $\alpha = \dfrac{37°}{2}$

$\therefore\ m\angle A = 2\alpha = 37°$

Clave E

22.



$\dfrac{7m}{7km} = \dfrac{2b}{b} \Rightarrow k = \dfrac{1}{2}$

$x = 7k + 9k = 16k$

$\therefore x = 16\left(\dfrac{1}{2}\right) = 8$

Clave D

23. Del gráfico:

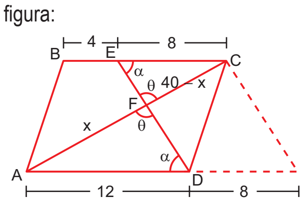
$$\frac{AC}{AB}=\frac{DF}{DE}$$

$$\Rightarrow \underbrace{AC \times DE}_{36} = \underset{x}{\underset{\downarrow}{AB}} \times \underset{3}{\underset{\downarrow}{DF}}$$

Luego: $x = 12$

Clave A

24. De la figura:



Entonces:

$$\frac{x}{40-x}=\frac{12}{8} \Rightarrow x = 24\text{ m}$$

Clave E

Nivel 3 (página 64) Unidad 3

Comunicación matemática

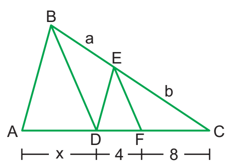
25.

26.

27.

Razonamiento y demostración

28.

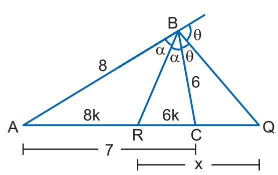


$$\frac{12}{x}=\frac{b}{a}=\frac{8}{4}$$

$\therefore x = 6$

Clave C

29.



$$14k = 7 \Rightarrow \quad k=\frac{1}{2}$$

$$\frac{x+8k}{x-6k}=\frac{8}{6} \Rightarrow \frac{x+4}{x-3}=\frac{4}{3}$$

$\therefore x = 24$

Clave D

30. $\dfrac{a}{3a}=\dfrac{6}{x} \Rightarrow x = 18$

Clave D

31. De la figura, planteamos:

$$\frac{AB}{BC}=\frac{AF}{FE} \quad \Rightarrow \frac{5}{3}=\frac{AF}{FE}$$

$$\frac{AC}{CD}=\frac{AF}{FE} \quad \Rightarrow \frac{AC}{CD}=\frac{5}{3}$$

Pero: $AC = 8 \Rightarrow \dfrac{8}{CD}=\dfrac{5}{3} \Rightarrow CD = 4{,}8$

Clave A

32. Del gráfico:

$$\frac{AC}{PR}=\frac{AB}{PQ}=\frac{BC}{QR} \qquad ...(1)$$

$3AC = 2PR$ (dato)

$$\frac{AC}{PR}=\frac{2}{3} \qquad ...(2)$$

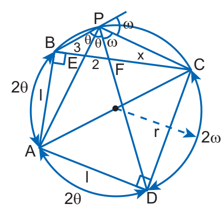
De(1) y (2):

$$\frac{AB}{PQ}+\frac{BC}{QR}=\frac{2}{3}+\frac{2}{3}=\frac{4}{3}$$

Clave C

Resolución de problemas

33.



Por el teorema de la cuaterna:

$$\frac{3}{2}=\frac{3+2+x}{x} \Rightarrow x = 10$$

Clave D

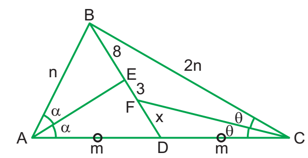
34. Por el teorema de la bisectriz:

$$\frac{x}{5}=\frac{7k}{5k}$$

$x = 7$

Clave A

35.



En el $\triangle ABD$ por el teorema de la bisectriz interior:

$$\frac{m}{n}=\frac{x+3}{8} \qquad ...(1)$$

En el $\triangle BCD$ por el teorema de la bisectriz interior:

$$\frac{m}{2n}=\frac{x}{11} \Rightarrow \frac{m}{n}=\frac{2x}{11} \qquad ...(2)$$

De (1) y (2):

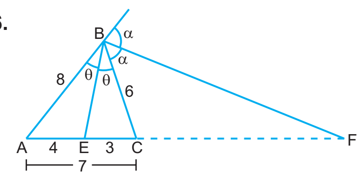
$$\frac{x+3}{8}=\frac{2x}{11} \Rightarrow 11x + 33 = 16x$$

$33 = 5x$

$\therefore x = 6{,}6$

Clave E

36.



En el $\triangle ABC$ por el teorema de la bisectriz interior:

$$\frac{AB}{BC}=\frac{AE}{EC} \Rightarrow \frac{AE}{EC}=\frac{8}{6}=\frac{4}{3}=k$$

$\Rightarrow AE = 4k \quad \wedge \quad EC = 3k$

Del gráfico: $AC = 7$

$7k = 7 \Rightarrow k = 1$

Luego: $AE = 4 \quad \wedge \quad EC = 3$

En el $\triangle ABC$ por el teorema de la bisectriz exterior:

$$\frac{8}{6}=\frac{7+CF}{CF} \quad \Rightarrow 8CF = 42 + 6CF$$

$2CF = 42$

$CF = 21$

Piden:

$EF = EC + CF = 3 + 21$

$\therefore EF = 24$

Clave C

SEMEJANZA DE TRIÁNGULOS

APLICAMOS LO APRENDIDO (página 66) Unidad 3

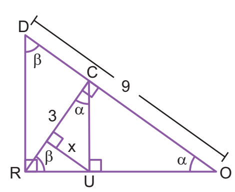
1. Por semejanza de triángulos:

$$\frac{4}{3}=\frac{m}{2} \quad \Rightarrow \quad \frac{(4)(2)}{3}=m$$

Por lo tanto: $m=\dfrac{8}{3}$

Clave E

2.



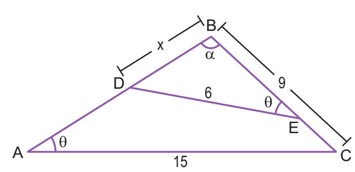
$\triangle DRO \sim \triangle RUC$

$$\Rightarrow \frac{3}{9}=\frac{x}{3}$$

$\therefore \; x = 1$

Clave D

3.



$\triangle ABC \sim \triangle DBE$

$$\frac{9}{15}=\frac{x}{6}$$

$\dfrac{(9)(6)}{15} = x$

$\therefore\ x = 3{,}6$

Clave A

4. Por propiedad:

$x = \dfrac{3\,.\,9}{3+9}$

$x = \dfrac{27}{12}$

Por lo tanto:
$x = 2{,}25$

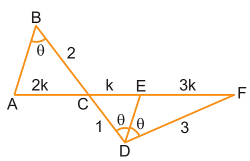
Clave A

5. Por el teorema de la bisectriz interior:

$\dfrac{CD}{CE} = \dfrac{DF}{EF} \Rightarrow \dfrac{1}{CE} = \dfrac{3}{EF}$

$\Rightarrow EF = 3CE$

Se observa:



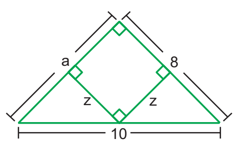
$\triangle ABC \sim \triangle CDE$

$\Rightarrow \dfrac{AC}{2} = \dfrac{CE}{1} \Rightarrow AC = 2CE$

Si: $CE = k$; $AC = 2k$ y $EF = 3k$
Como: $AF = 6$
$\Rightarrow 6k = 6$
$k = 1$
$\therefore\ CE = 1$

Clave A

6.



Calculando a por el teorema de Pitágoras:
$10^2 = 8^2 + a^2$
$36 = a^2 \Rightarrow a = 6$

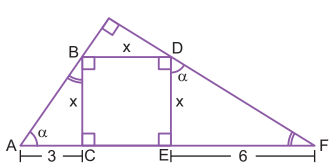
Por propiedad:

$\dfrac{(8)(a)}{8+a} = \dfrac{(8)(6)}{8+6}$

$\therefore z = \dfrac{24}{7}$

Clave B

7. De la figura:



$\triangle ABC \sim \triangle DFE$:

$\dfrac{x}{3} = \dfrac{6}{x} \Rightarrow x^2 = 18$

$\therefore x = 3\sqrt{2}$

Clave A

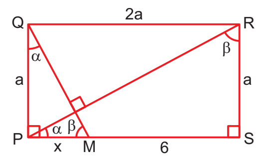
8. Por propiedad:

$\dfrac{1}{x} = \dfrac{1}{8} + \dfrac{1}{3}$

$\dfrac{1}{x} = \dfrac{11}{24} \Rightarrow x = \dfrac{24}{11}$

Clave B

9. Del enunciado:



$\triangle QPM \sim \triangle PSR$

$\dfrac{x}{a} = \dfrac{a}{x+6} \Rightarrow x(x+6) = a^2$ ...(1)

Pero: $x + 6 = 2a$ ...(2)

Reemplazando (2) en (1):

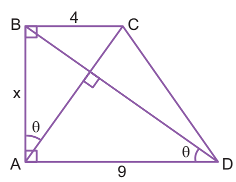
$\Rightarrow x(x+6) = \left(\dfrac{x+6}{2}\right)^2$

$x(x+6) = \dfrac{(x+6)(x+6)}{4}$

$4x = x + 6$
$3x = 6 \Rightarrow x = 2$

Clave D

10.

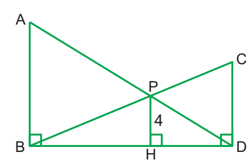


$\triangle DAB \sim \triangle ABC$

$\dfrac{x}{9} = \dfrac{4}{x}$
$x^2 = 36$
$x = 6$

Clave B

11.

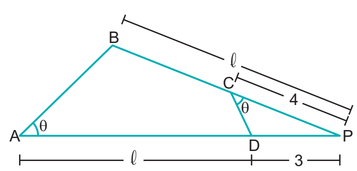


Por propiedad:

$\dfrac{1}{AB} + \dfrac{1}{CD} = \dfrac{1}{4} = 0{,}25$

Clave A

12.



$\triangle ABP \sim \triangle CDP$

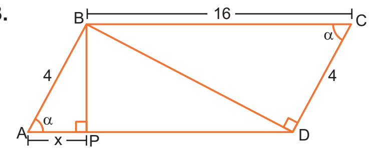
$\dfrac{CP}{AP} = \dfrac{DP}{BP}$

$\dfrac{4}{\ell+3} = \dfrac{3}{\ell}$

$\therefore\ \ell = 9$

Clave A

13.



$\triangle ABP \sim \triangle BDC$

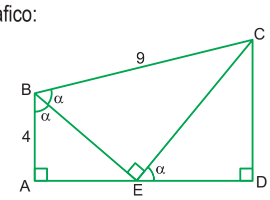
$\dfrac{AP}{DC} = \dfrac{AB}{BC}$

$\dfrac{x}{4} = \dfrac{4}{16}$

$\therefore\ x = 1$

Clave E

14. Del gráfico:



$m\angle ABE = m\angle CED$

$\Rightarrow \triangle BAE \sim \triangle BEC$

$\dfrac{BA}{BE} = \dfrac{BE}{BC}$

$(BE)^2 = (BA)(BC)$

$(BE)^2 = (4)(9)$

$\therefore\ BE = 6$

Clave D

Practiquemos

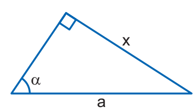
Nivel 1 (página 68) Unidad 3

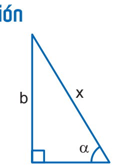
Comunicación matemática

1.
2.
3.

Razonamiento y demostración

4.



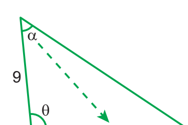


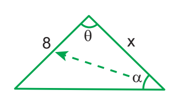
$\dfrac{x}{a} = \dfrac{b}{x} \Rightarrow x^2 = ab$

Como: $ab = 48$
$x^2 = 48$
$x = 4\sqrt{3}$

Clave C

5.





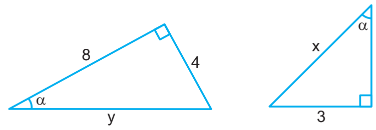
$\dfrac{2x}{9} = \dfrac{8}{x} \Rightarrow 2x^2 = 72$

$x^2 = 36$

$x = 6$

Clave D

6.



$y^2 = 8^2 + 4^2$

$y^2 = 64 + 16$

$y^2 = 80$

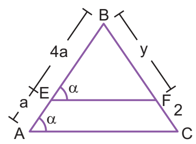
$\Rightarrow y = 4\sqrt{5}$

$\dfrac{4}{y} = \dfrac{3}{x} \Rightarrow x = \dfrac{3y}{4}$

$x = \dfrac{3(4\sqrt{5})}{4} = 3\sqrt{5}$

Clave E

7.



$\dfrac{4a}{a} = \dfrac{y}{2}$

$y = 8$

Piden: BC

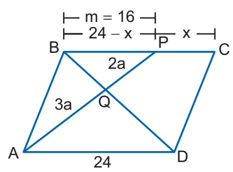
$BC = y + 2$

$\therefore\ BC = 10$

Clave B

Resolución de problemas

8.



$\Delta BPQ \sim \Delta DAQ$

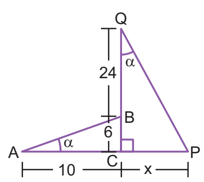
$\dfrac{m}{2a} = \dfrac{24}{3a} \Rightarrow m = 16$

$24 - x = 16$

$x = 8$

Clave D

9.



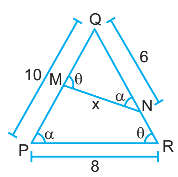
$\Delta ACB \sim \Delta QCP$

$\dfrac{10}{6} = \dfrac{30}{x}$

$x = 18$

Clave E

10.



$\Delta PQR \sim \Delta NQM$

$\dfrac{x}{8} = \dfrac{6}{10}$

$x = 4{,}8$

Clave C

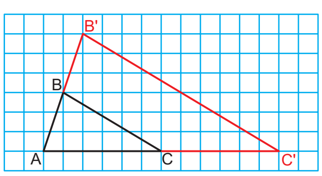
Nivel 2 (página 69) Unidad 3

Comunicación matemática

11.

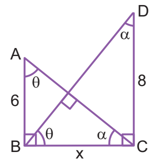
12.

13.



Razonamiento y demostración

14.



$\Delta ABC \sim \Delta BCD$

$\dfrac{x}{6} = \dfrac{8}{x}$

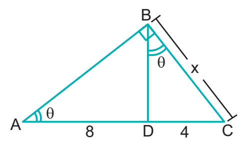
$x\,(x) = 6(8)$

$x^2 = 48$

$x = 4\sqrt{3}$

Clave B

15.



$\Delta ABC \sim \Delta BDC$

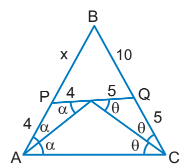
$\dfrac{12}{x} = \dfrac{x}{4}$

$x^2 = 48$

$x = 4\sqrt{3}$

Clave B

16.

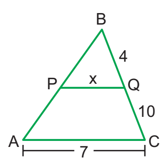


$\dfrac{x}{4} = \dfrac{10}{5}$

$\Rightarrow x = 8$

Clave D

17.

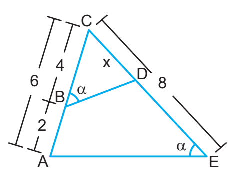


$\dfrac{14}{7} = \dfrac{4}{x}$

$\Rightarrow x = 2$

Clave B

18.



$\Delta ACE \sim \Delta DCB$

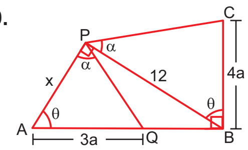
$\dfrac{x}{4} = \dfrac{6}{8}$

$x = 3$

Clave C

Resolución de problemas

19.



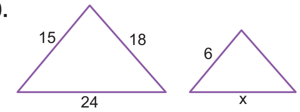
$\Delta APQ \sim \Delta BPC$

$\dfrac{x}{3a} = \dfrac{12}{4a}$

$x = 9$

Clave D

20.

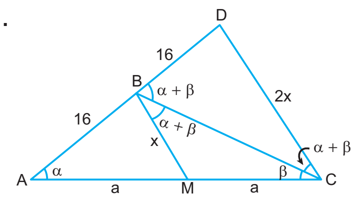


$\dfrac{15}{24} = \dfrac{6}{x}$

$\Rightarrow\ x = 9{,}6$

Clave B

21.



Trazamos $\overline{DC} \,//\, \overline{BM}$

$\Rightarrow 2x = 16$

$\therefore\ x = 8$

Clave C

Nivel 3 (página 70) Unidad 3

Comunicación matemática

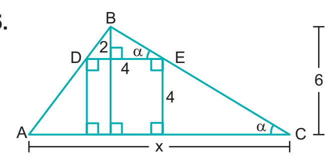
22.

23.

24.

Razonamiento y demostración

25.



$\Delta ABC \sim \Delta DBE$

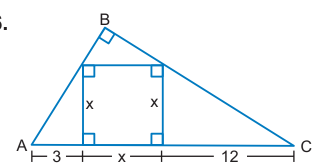
$\dfrac{6}{x} = \dfrac{2}{4}$

$24 = 2x$

$\Rightarrow x = 12$

Clave E

26.



Por propiedad:

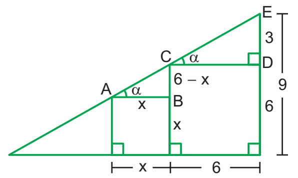
$x = \sqrt{3 \times 12}$

$x = \sqrt{36}$

$\Rightarrow\ x = 6$

Clave C

27.



$\Delta ABC \sim \Delta CDE$

$\frac{6-x}{x}=\frac{3}{6}$

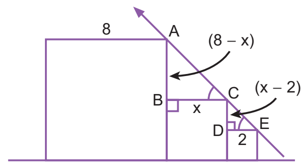
$36-6x=3x$

$36=9x$

$x=4$

Clave D

28. De la figura:



$\Delta ABC \sim \Delta CDE$

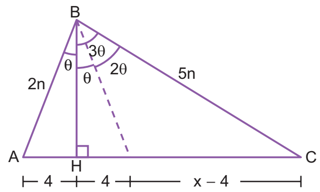
$\frac{8-x}{x-2}=\frac{x}{2} \Rightarrow 16-2x=x^2-2x$

$\therefore x=4$

Clave B

Resolución de problemas

29. Según el enunciado:



$\frac{2n}{8}=\frac{5n}{x-4} \Rightarrow 2(x-4)=5(8)$

$2x-8=40$

$2x=48$

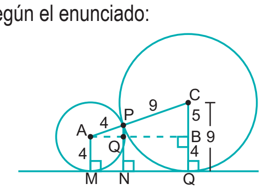
$x=24$

Luego:

$\frac{HC}{8}=\frac{24}{8}=3$

Clave B

30. Según el enunciado:



$\Delta AQP \sim \Delta ABC$; $PN = x$

$\frac{PQ}{CB}=\frac{AP}{AC}$

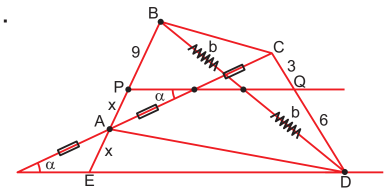
$\frac{x-4}{9-4}=\frac{4}{13} \Rightarrow 13x-52=20$

$13x=72$

$\therefore x=\frac{72}{13}$

Clave A

31.

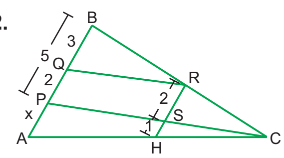


$\overline{PQ} \parallel \overline{ED}$

$\Rightarrow \frac{9}{2x}=\frac{b}{b} \quad \Rightarrow x=4,5$

Clave C

32.



$\Delta APC \sim \Delta HSC \ \wedge \ \Delta PBC \sim \Delta SRC$

$\frac{5}{x}=\frac{2}{1}$

$5=2x$

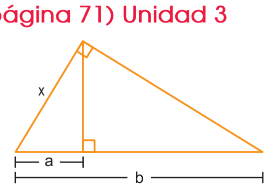
$x=2,5$

Clave C

RELACIONES MÉTRICAS EN EL TRIÁNGULO RECTÁNGULO

APLICAMOS LO APRENDIDO (página 71) Unidad 3

1.



Sabemos:

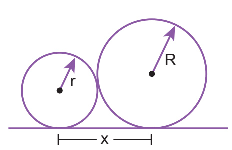
$x^2=ab$

En el problema:

$x^2=1(4) \Rightarrow x=2$

Clave B

2.



Sabemos:

$x=2\sqrt{Rr}$

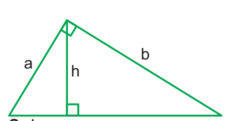
En el problema:

$x=2\sqrt{\sqrt{2}\times 2\sqrt{2}}$

$\therefore \ x=4$

Clave C

3.



Sabemos:

$\frac{1}{h^2}=\frac{1}{a^2}+\frac{1}{b^2}$

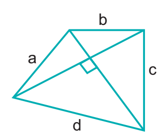
En el problema:

$\frac{1}{x^2}=\frac{1}{3^2}+\frac{1}{4^2}=\frac{16+9}{(9)(16)}$

$x^2=\frac{144}{25} \Rightarrow x=2,4$

Clave B

4.



Sabemos:

$a^2+c^2=b^2+d^2$

En el problema:

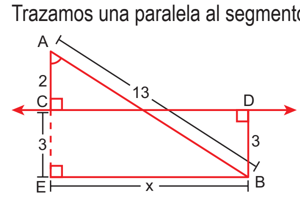
$x^2+(\sqrt{8})^2=(\sqrt{6})^2+(\sqrt{7})^2$

$x^2+8=6+7$

$x^2=5 \Rightarrow x=\sqrt{5}$

Clave C

5. Trazamos una paralela al segmento CD:



Por el teorema de Pitágoras:

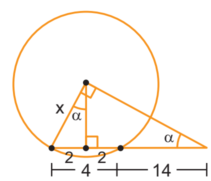
$13^2=5^2+x^2$

$169=25+x^2$

$144=x^2 \Rightarrow \ x=12$

Clave A

6.



Por propiedad:

$x^2=2(2+2+14)$

$x^2=2(18) \Rightarrow x^2=36$

$\therefore \ x=6$

Clave E

7. Por propiedad:

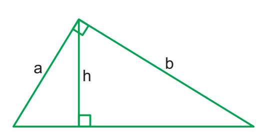
$12^2=x(16)$

$144=16x$

$\frac{144}{16}=x \Rightarrow x=9$

Clave C

8.



Sabemos:

$$\frac{1}{h^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

En el problema:

$$\frac{1}{x^2}=\frac{1}{5^2}+\frac{1}{12^2}\Rightarrow\frac{1}{x^2}=\frac{144+25}{(25)(144)}$$

$$x^2=\frac{(25)(144)}{169}\Rightarrow x=\sqrt{\frac{(25)(144)}{169}}$$

$$x=\frac{(5)(12)}{13}\qquad\therefore x=\frac{60}{13}$$

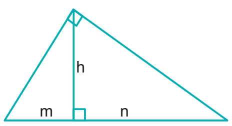
Clave A

9. Por propiedad:

$6^2=2(x+2)$
$36=2(x+2)$
$18=x+2\Rightarrow x=16$

Clave E

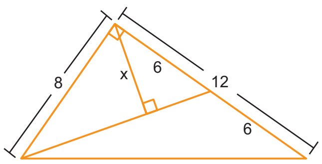
10.



Sabemos:
$h^2=mn$
En el problema:
$(\sqrt{8})^2=(x+1)(x-1)$
$8=x^2-1\Rightarrow x^2=9$
$\therefore x=3$

Clave C

11. Piden: x



Por propiedad:

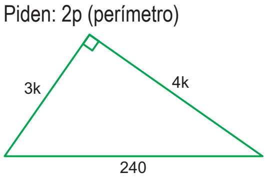
$$\frac{1}{x^2}=\frac{1}{8^2}+\frac{1}{6^2}$$

$$\frac{1}{x^2}=\frac{1}{64}+\frac{1}{36}\Rightarrow x^2=\frac{(64)(36)}{100}$$

Resolviendo:
$\therefore x=4{,}8$

Clave E

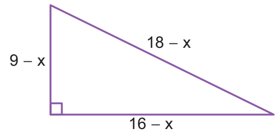
12. Piden: 2p (perímetro)



Por Pitágoras:
$9k^2+16k^2=57\,600$
$25k^2=57\,600$
$k^2=2304$
$k=48$
$\therefore 2p=3k+4k+240=576$

Clave C

13.

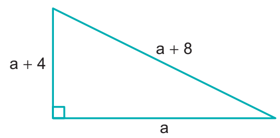


Por Pitágoras:
$(18-x)^2=(9-x)^2+(16-x)^2$
$324-36x+x^2=81-18x+x^2+256-32x+x^2$
$0=13+x^2-14x$
$0=x^2-14x+13$
$0=(x-13)(x-1)$

$x-13=0\quad\vee\quad x-1=0$
$x=13$ (no cumple) $x=1$(si cumple)
$\therefore x=1$

Clave E

14.



Por Pitágoras:
$a^2+(a+4)^2=(a+8)^2$
$a^2+a^2+8a+16=a^2+16a+64$
$a^2-8a-48=0$
$(a-12)(a+4)=0$

$a-12=0\quad\vee\quad a+4=0$
$a=12\qquad a=-4$ (no cumple)

Por lo tanto:
La hipotenusa mide: $a+8=20$

Clave A

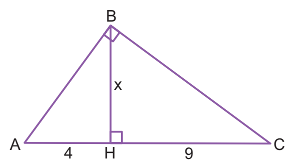
Practiquemos
Nivel 1 (página 73) Unidad 3

Comunicación matemática

1.
2.
3.

Razonamiento y demostración

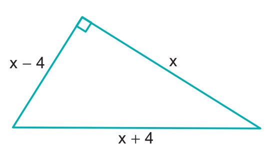
4.



$x^2=9\times 4$
$x^2=36\Rightarrow x=6$

Clave C

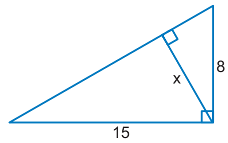
5.



$x^2+8x+16=x^2+x^2-8x+16$
$0=x^2-16x$
$x=16\quad\vee\quad x=0$
$\Rightarrow x=16$

Clave E

6. Piden: x



Por propiedad:

$$\frac{1}{x^2}=\frac{1}{64}+\frac{1}{225}$$

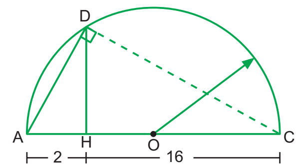
$(64)(225)=289x^2$

$$\frac{(64)(225)}{289}=x^2$$

$$\therefore\ x=\frac{120}{17}$$

Clave B

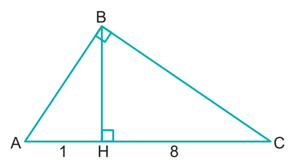
7. Piden: AD



Completamos el gráfico.
Por propiedad:
$(AD)^2=18(2)$
$\therefore AD=6$

Clave A

8.

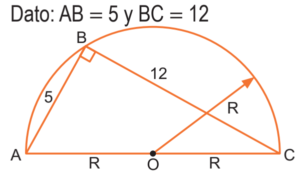


$(AB)^2=(1)(9)$
$AB=3$

Clave C

9. Piden: R

Dato: $AB=5$ y $BC=12$

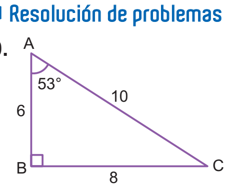


Del gráfico, por Pitágoras:
$5^2+12^2=(2R)^2$
$169=4R^2$
$\therefore\ R=6{,}5$

Clave C

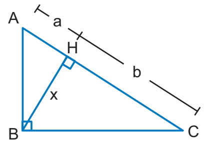
Resolución de problemas

10.



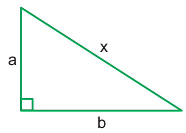
$\overline{BC}$ es proyección de $\overline{AC}$ sobre el cateto mayor.
Por lo tanto, la medida de la proyección de la hipotenusa sobre el cateto mayor es: 8

Clave B

11. 

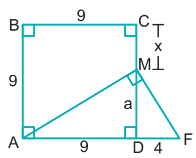
Dato: $ab = 49$
Por relaciones métricas:
$x^2 = ab$
$x^2 = 49 \Rightarrow x = 7$

Clave A

12. 

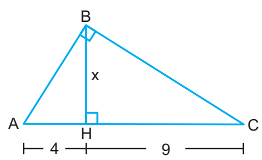
Dato:
$a^2 + b^2 + x^2 = 288 \Rightarrow a^2 + b^2 = 288 - x^2$
Piden x
Por El teorema de Pitagoras
$x^2 = \underbrace{a^2 + b^2}_{288 - x^2}$
Entonces:
$2x^2 = 288$
$x^2 = 144$
$x = 12$

Clave C

13. 

$a^2 = 9 \times 4$
$a = 6$
Como ABCD es un cuadrado:
$a + x = 9 \Rightarrow x = 3$

Clave C

14. 

$x^2 = 4 \times 9$
$\Rightarrow x = 6$

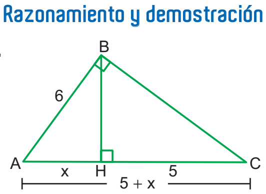
Clave C

Nivel 2 (página 74) Unidad 3

Comunicación matemática

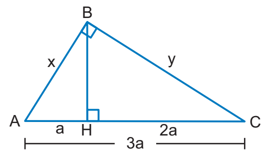
15.
16.
17.

Razonamiento y demostración

18. 

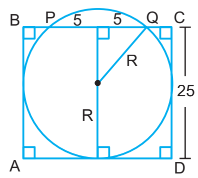
$6^2 = x(5 + x)$
$6^2 = 5x + x^2$
$x^2 + 5x - 36 = 0$
$x = 4 \quad \vee \quad x = -9$ (no cumple)
$\Rightarrow x = 4$

Clave C

19. 

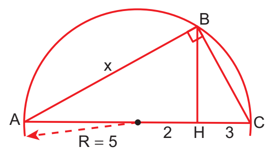
$x^2 = 3a(a) \Rightarrow x = a\sqrt{3}$
$y^2 = 3a(2a) \Rightarrow y = a\sqrt{6}$
Piden: $\dfrac{x}{y} = \dfrac{\sqrt{3}}{\sqrt{6}}$
$\Rightarrow \dfrac{x}{y} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

Clave A

20. 

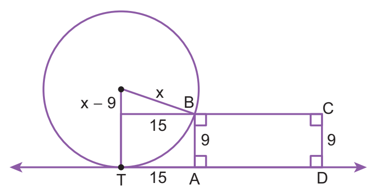
Por el teorema de Pitágoras
$R^2 = (25 - R)^2 + 5^2$
$R^2 = 625 - 50R + R^2 + 25$
$50R = 650 \Rightarrow R = 13$

Clave C

21. 

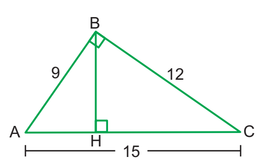
$x^2 = 10(7)$
$x = \sqrt{70}$

Clave B

22. 

$x^2 = (x - 9)^2 + 15^2$
$18x = 306$
$x = 17$

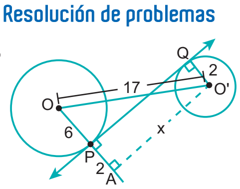
Clave E

23. 

$9(12) = 15(BH)$
$BH = 7{,}2$ m

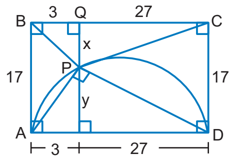
Clave A

Resolución de problemas

24. 

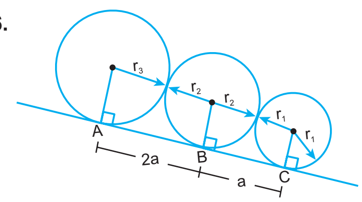
Por el teorema de Pitágoras en el ⊿AOO':
$17^2 = x^2 + 8^2$
$x = 15$

Clave C

25. 

En el ΔAPD se cumple:
$y^2 = 27(3)$
$y^2 = 81 \Rightarrow y = 9$
Pero:
$x + y = 17$
$\therefore x = 8$

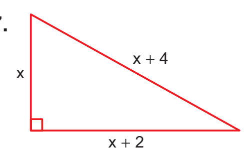
Clave A

26. 

$\left.\begin{aligned} 2\sqrt{r_3 r_2} &= 2a \\ 2\sqrt{r_1 r_2} &= a \end{aligned}\right\} \dfrac{r_3}{r_1} = \dfrac{4}{1}$

Piden: $\dfrac{r_1}{r_3} = \dfrac{1}{4}$

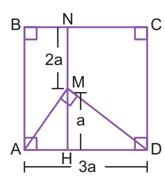
Clave C

27. 

Sea x un número par.
Aplicando el teorema de Pitágoras:
$x^2 + (x + 2)^2 = (x + 4)^2$
$x^2 + x^2 + 4x + 4 = x^2 + 8x + 16$
$x^2 - 4x - 12 = 0$
$(x - 6)(x + 2) = 0$
$x - 6 = 0 \quad \vee \quad x + 2 = 0$
$x = 6 \qquad x = -2$
(no cumple)
Por lo tanto el menor lado es: 6

Clave B

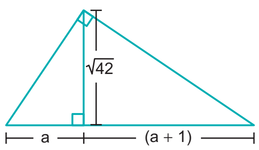
28. Piden: 2p (perímetro del cuadrado)
Datos: MN = 2MH $\wedge$ (AM)(MD) = 12



Por propiedad:
$\Rightarrow \underbrace{(AM)(MD)}_{12} = (MH)(AD)$
$= a(3a) \Rightarrow a = 2$
$\therefore 2p = 4(3a) = 4(6) = 24$ cm

Clave C

29. Piden: hipotenusa



Por propiedad:

$(\sqrt{42})^2 = a(a+1)$

$42 = a(a+1)$

$6(7) = a(a+1)$

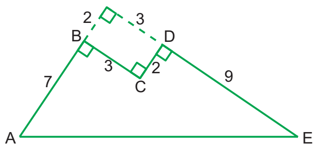
$\Rightarrow a = 6$

Por lo tanto la hipotenusa es:

$2a + 1 = 2(6) + 1 = 13$

Clave A

30.



$(AE)^2 = 9^2 + 12^2$

$AE = 15$

Clave E

Nivel 3 (página 75) Unidad 3

Comunicación matemática

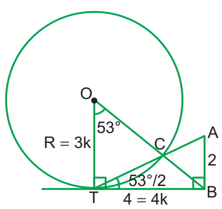
31.

32.

33.

Razonamiento y demostración

34.

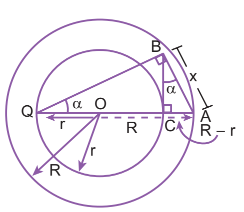


$k = 1 \ \wedge \ R = 3k$

$\Rightarrow R = 3$

Clave B

35.



Dato: $R^2 - r^2 = 25$

Piden x:

$x^2 = (r+R)(R-r)$

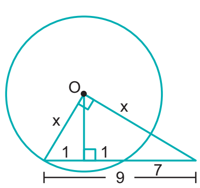
$x^2 = R^2 - r^2$

$x^2 = 25$

$x = 5$

Clave D

36.

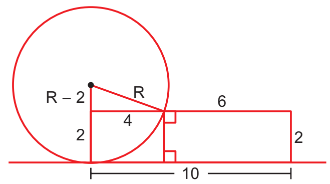


$x^2 = 1(9)$

$\Rightarrow x = 3$

Clave E

37.



$R^2 = (R-2)^2 + 4^2$

$R^2 = R^2 - 4R + 4 + 16$

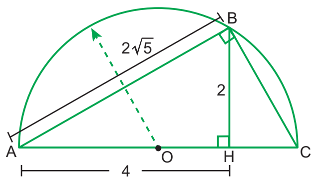
$4R = 20$

$R = 5$

Clave E

38. Piden: HC

Datos: $AB = 2\sqrt{5}$ y $AH = 4$



Por el teorema de Pitágoras:

$(2\sqrt{5})^2 = 16 + (BH)^2$

$BH = 2$

Por propiedad:

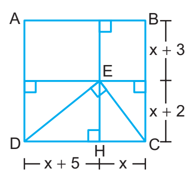
$(BH)^2 = (AH)(HC)$

$2^2 = 4(HC)$

$\therefore HC = 1$

Clave B

39. Piden: x



Por propiedad:

$(EH)^2 = (DH)(HC)$

$(x+2)^2 = (x+5)x$

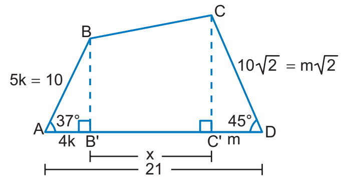
$x^2 + 4x + 4 = x^2 + 5x$

$\therefore x = 4$

Clave C

Resolución de problemas

40.



$k = 2 \ \wedge \ m = 10$

$\overline{B'C'}$ es la proyección de $\overline{BC}$ sobre $\overline{AD}$:

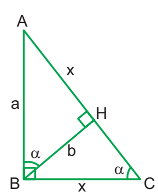
$4k + x + m = 21$

$4(2) + x + 10 = 21$

$\Rightarrow x = 3$

Clave B

41.



Se cumple:

$\frac{1}{b^2} = \frac{1}{a^2} + \frac{1}{x^2} \ \wedge \ a^2 = b^2 + x^2$

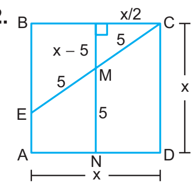
$\frac{1}{b^2} - \frac{1}{a^2} = \frac{1}{x^2} \Rightarrow \frac{a^2 - b^2}{(ab)^2} = \frac{1}{x^2}$

$x^4 = 16^2$

$x = 4$

Clave B

42.



Por el teorema de Pitágoras:

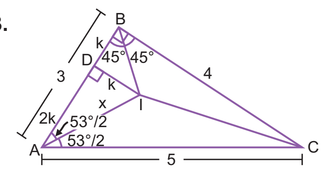
$5^2 = (x-5)^2 + \left(\frac{x}{2}\right)^2$

$25 = x^2 - 10x + 25 + \frac{x^2}{4}$

$10x = \frac{5x^2}{4} \Rightarrow x = 8$

Clave B

43.



$2k + k = 3 \Rightarrow k = 1$

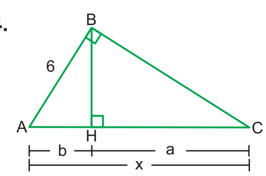
En $\triangle ADI$:

$x^2 = (2k)^2 + k^2 = 2^2 + 1^2$

$x^2 = 5 \Rightarrow x = \sqrt{5}$

Clave C

44.



Dato: $\left.\begin{array}{l} a - b = 1 \\ a + b = x \end{array}\right\} (+)$

$\Rightarrow a = \frac{x+1}{2} \ \wedge \ b = \frac{x-1}{2}$

Además: $6^2 = bx$

$36 = \left(\frac{x-1}{2}\right)x$

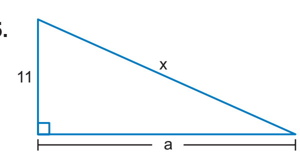
$36 = \frac{x^2 - x}{2}$

$72 = x^2 - x \Rightarrow 9(8) = (x)(x-1)$

$x = 9$

Clave E

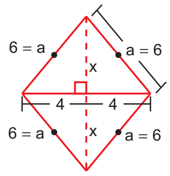
45.



$$x^2 = 11^2 + a^2$$
$$(x + a)(x - a) = 121(1)$$
$$\left.\begin{aligned} x + a &= 121 \\ x - a &= 1 \end{aligned}\right\} (+)$$
$$2x = 122$$
$$\Rightarrow x = 61 \quad \wedge \quad a = 60$$

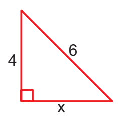
Clave C

46. Piden: longitud de la otra diagonal



Dato: $2p = 24$
Como:
$2p = 4a = 24$
$\Rightarrow a = 6$

Por el teorema de Pitágoras:



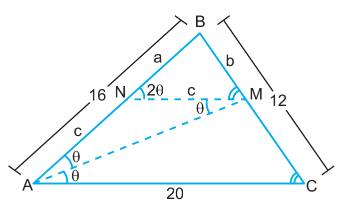
$$\Rightarrow x^2 + 16 = 36$$
$$x^2 = 20$$
$$\Rightarrow x = 2\sqrt{5}$$
$$\therefore 2x = 4\sqrt{5}$$

Clave C

MARATÓN MATEMÁTICA (página 77)

1. Como $\overline{AC}$ es paralelo a $\overline{MN}$, entonces afirmamos que $\Delta ABC \sim \Delta NBM$ por lo tanto se cumple:

$$\frac{a}{16} = \frac{b}{12} = \frac{c}{20} = k \text{ ; despejando en función de k}$$



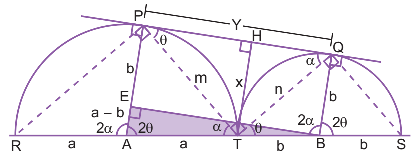
$a = 16k$, $b = 12k$ y $c = 20k$
Sin embargo el ΔANM es isósceles ya que $m\angle NAM = m\angle NMA = \theta$
$\therefore\ a + c = 16$

Reemplazando:
$16k + 20k = 16 \Rightarrow 36k = 16$
$\therefore\ k = 4/9$, reemplazando en las proporciones:
$a = 64/9$; $b = 48/9$ y $c = 80/9$

Luego
$$2P_{\Delta MBN} = a + b + c$$
$$\Rightarrow 2P_{\Delta MBN} = \frac{64}{9} + \frac{48}{9} + \frac{80}{9} = \frac{64}{3}$$

Clave E

2. Trazamos $\overline{HT}$, el cual es la distancia de T hacia $\overline{PQ}$; luego trazamos $\overline{RP}$; $\overline{PT}$; $\overline{QT}$ y $\overline{QS}$, formando así los triángulos rectángulos RPT; PTQ y TQS.



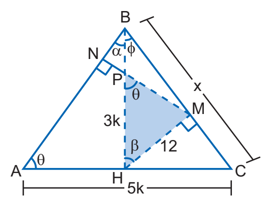
Luego vemos que ⊿RPT ∼ ⊿PTQ
$$\Rightarrow \frac{m}{2a} = \frac{x}{m} \Rightarrow m^2 = 2ax$$
también vemos que ⊿TQS ∼ ⊿PTQ
$$\Rightarrow \frac{n}{2b} = \frac{x}{n} \Rightarrow n^2 = 2bx$$
Por el teorema de Pitágoras:
$$y^2 = m^2 + n^2 = 2x(a + b) \qquad \text{... (I)}$$

Finalmente trazamos $\overline{BE}$ paralela a $\overline{PQ}$; formando el triángulo rectángulo BEA y en donde aplicamos el teorema de Pitágoras:
$$(a - b)^2 + y^2 = (a + b)^2$$

Reemplazando de (I)
$$y^2 = (a + b)^2 - (a - b)^2 = 2x(a + b)$$
$$(a + b - a + b)(a + b + a - b) = 2x(a + b)$$
$$\Rightarrow x = \frac{2ab}{(a + b)}$$

Clave D

3. Vemos que $m\angle BAC = m\angle HPM = \theta$ pues $\theta + \alpha = 90°$; asimismo vemos que

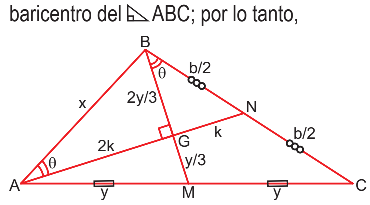


Dato:
$$\frac{AC}{5} = \frac{PH}{3} = k$$

$m\angle BHM = m\angle BCA = \beta$; pues $\beta + \phi = 90°$
$\therefore\ \Delta ABC \sim \Delta PMH$
$$\Rightarrow \frac{5k}{3k} = \frac{x}{12} \Rightarrow x = 20$$

Clave C

4. Como $\overline{AN}$ y $\overline{BM}$ son medianos, entonces G es el baricentro del ⊿ABC; por lo tanto,

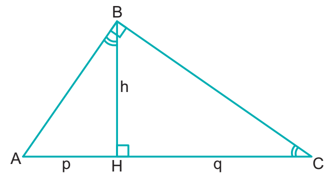


G divide a dichas medianas en segmentos que están en la razón de 2 a 1 $\Rightarrow AG = 2k$ y $GN = k$
Luego: $BM = MC = AM = y$ pues $\overline{BM}$ es la mediana relativa a la hipotenusa luego vemos que ⊿AGB ∼ ⊿BGN
$$\Rightarrow \frac{2y/3}{x} = \frac{k}{b/2} \Rightarrow yb = 3kx \qquad \text{... (I)}$$
Ahora en el ⊿AGM aplicamos el teorema de Pitágoras:
$$y^2 = (2k)^2 + (y/3)^2$$

$$\frac{8y^2}{9} = 4k^2 \Rightarrow y\sqrt{2} = 3k\text{; reemplazamos en (I)}$$
$$\therefore y\,b = y\sqrt{2}\,x \Rightarrow x = \frac{\sqrt{2}}{2}b$$

Clave E

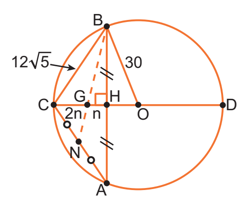
5. Sabemos que los triángulos rectángulos AHB y BHC son semejantes dado que el ⊿ABC es rectángulo; por lo tanto se cumple:



$h^2 = pq; \Rightarrow p = h^2/q \qquad \text{... (I)}$
dato: $q - p = h$, reemplazando en (I)
$$q - \frac{h^2}{q} = h$$
$$q^2 - hq = h^2 \quad \left(\text{sumamos } \frac{h^2}{4}\right)$$
$$q^2 - hq + \frac{h^2}{4} = \frac{5h^2}{4}$$
$$\left(q - \frac{h}{2}\right)^2 = \frac{5h^2}{4}$$
$$q = \frac{h}{2} + \frac{\sqrt{5}\,h}{2}$$
$$\Rightarrow q = \frac{h}{2}(\sqrt{5} + 1)\text{ ; reemplazando en (I):}$$
$$p = \frac{h}{2}(\sqrt{5} - 1)$$
Hipotenusa = p + q
$$\Rightarrow AC = \frac{h}{2}(\sqrt{5} + 1) + \frac{h}{2}(\sqrt{5} - 1)$$
$$\therefore\ AC = \sqrt{5}\,h$$

Clave C

6. Trazamos la mediana BN, la cual interseca al diámetro CD en G; dado que CH es mediana, entonces G es el baricentro luego sabemos que $CG = 2GM \Rightarrow$ denominamos $2n = CG$ y $n = GM$; también vemos que $CO = 30$ (radio)
$\Rightarrow\ HO = 30 - 3n$



Luego usamos el teorema de Pitágoras en el ⊿CBH:
$$(12\sqrt{5})^2 = (3n)^2 + BH^2 \qquad \text{... (I)}$$

Igualmente en el ⊿BHO:
$$30^2 = (30 - 3n)^2 + BH^2 \qquad \text{... (II)}$$

Restando (I) – (II)
$$(12\sqrt{5})^2 - 30^2 = (3n)^2 - (30 - 3n)^2$$
$$(12)(12)5 - (30)(30) = (3n + 30 - 3n)(3n - 30 + 3n)$$
$$9(4 \times 4 \times 5 - 10 \times 10) = 30 \times 3(2n - 10)$$
$$\Rightarrow\ 4 - 5 = n - 5 \Rightarrow n = 4$$
$$\therefore\ CG = 8 \text{ m}$$

Clave D

APLICAMOS LO APRENDIDO (página 80) Unidad 4

1. Observamos que es un triángulo equilátero, entonces:

$$S = \frac{a^2\sqrt{3}}{4}$$

$$S = \frac{6^2\sqrt{3}}{4}$$

$$\therefore S = 9\sqrt{3}\,m^2$$

Clave A

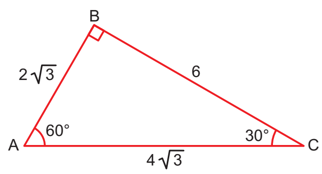
2. El área será:

$$\frac{bh}{2} = \frac{(4)(3)}{2} = \frac{12}{2}$$

Entonces:
Área = 6 cm^2

Clave D

3. Del triángulo:

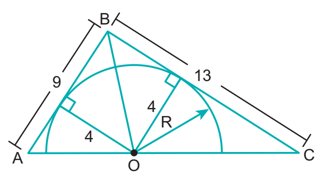


Por lo tanto:

$$\text{Área} = \frac{(2\sqrt{3})(6)}{2} = 6\sqrt{3}\ m^2$$

Clave D

4.



$$S_{\Delta ABC} = S_{\Delta AOB} + S_{\Delta BOC}$$

$$S_{\Delta ABC} = \frac{9(4)}{2} + \frac{13(4)}{2}$$

$$\therefore\ S_{\Delta ABC} = 18 + 26 = 44$$

Clave D

5. Sea S el área de la región sombreada, entonces:

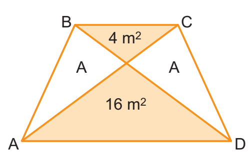
$$S = \left(\frac{8+4}{2}\right)5$$

$S = (6)5$

$\therefore S = 30$

Clave B

6. Se deduce que:



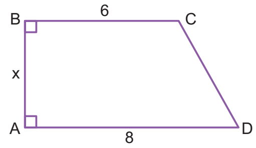
$A^2 = (4)(16) \Rightarrow A = 8$

Área total $= 4 + 16 + 2A$
$= 4 + 16 + 16 = 36$

Por lo tanto:
Área total es: 36 m^2

Clave B

7. Según el enunciado:

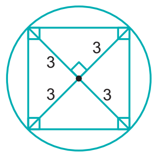


$$A = \left(\frac{6+8}{2}\right)x \quad \Rightarrow \quad 28 = \left(\frac{6+8}{2}\right)x$$

Resolviendo:
$28 = 7x \Rightarrow x = 4$ m

Clave C

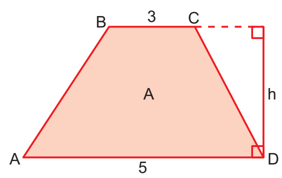
8. Según el enunciado:



El área del cuadrado es:
$(3\sqrt{2})^2 = 18\ cm^2$

Clave A

9.

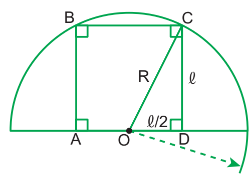


$$A = \left(\frac{3+5}{2}\right)h = 44$$

$$\therefore h = 11 \text{ m}$$

Clave C

10.

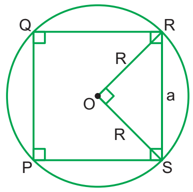


En el ◺ODC por el teorema de Pitágoras:

$$R^2 = \ell^2 + \frac{\ell^2}{4} \Rightarrow R^2 = \frac{5\ell^2}{4}$$

$$\Rightarrow\ \ell^2 = \frac{4R^2}{5}$$

Entonces: $A_{\square ABCD} = \dfrac{4R^2}{5}$



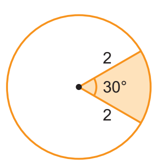
En el ◺ROS por el teorema de Pitágoras:
$a^2 = R^2 + R^2 \Rightarrow a^2 = 2R^2 \Rightarrow A_{\square PQRS} = 2R^2$

Piden:

$$\frac{A_{\square ABCD}}{A_{\square PQRS}} = \frac{\frac{4R^2}{5}}{2R^2} = \frac{2}{5}$$

Clave B

11.

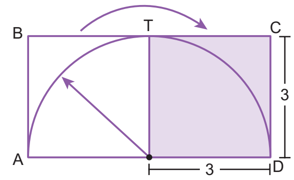


$$A = \frac{30°\pi(2)^2}{360°}$$

$$\therefore A = \frac{\pi}{3}\ cm^2$$

Clave B

12. Toda la zona sombreada equivale a un cuadrado de lado 3.



$\Rightarrow A = 3^2 = 9\ cm^2$

Clave C

13. $A = A_{\blacksquare} - A_{◣}$

$$A = 2^2 - \frac{90°\pi(2)^2}{360°}$$

$$\therefore A = 4 - \pi$$

Clave D

14.

$A = A_{\square} - A_{⌓}$

$$A = 4^2 - \frac{\pi(2)^2}{2}$$

$\therefore A = 16 - 2\pi$

Clave E

PRACTIQUEMOS

Nivel 1 (página 82) Unidad 4

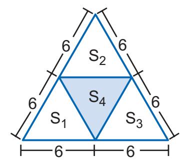
Comunicación matemática

1.

2.

Razonamiento y demostración

3.



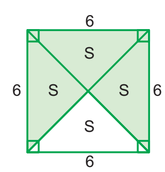
$$S_{\triangle} = \frac{12^2\sqrt{3}}{4} = 36\sqrt{3}$$

$$S_1 = S_2 = S_3 = S_4 = \frac{S_{\triangle}}{4}$$

Piden S_4:
$S_4 = 9\sqrt{3}$

Clave D

4.



$4S = 6^2 \ \Rightarrow\ S = 9$
Piden: $3S = 27$

Clave B

5. Se deduce que:
QO = 1 (radio)
PR = $\sqrt{3}$ (Δ notable)
m∠POR = 120°

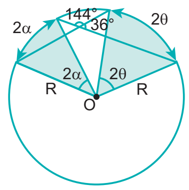
$A_{pedida} = A_{\odot} - A_{\triangleleft POR}$

$$A_{pedida} = \pi(1)^2 - \frac{120°\pi(1)^2}{360°}$$

$$A_{pedida} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Clave C

6.



$\frac{2\theta + 2\alpha}{2} = 36° \Rightarrow \theta + \alpha = 36°$

$A = \frac{2\alpha\pi 6^2}{360°} + \frac{2\theta\pi 6^2}{360°} = \frac{2(\theta+\alpha)\pi 36}{360°}$

$\therefore A = (2)\frac{36°\pi}{10°} = \frac{36\pi}{5}$

Clave D

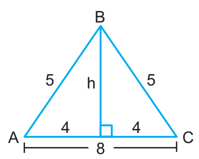
Resolución de problemas

7. Los lados del triángulo son radios de los cuartos de circunferencia.

$\therefore A_{\triangle} = \frac{L^2\sqrt{3}}{4} = \frac{4^2\sqrt{3}}{4} = 4\sqrt{3}\ m^2$

Clave D

8.

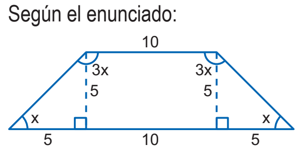


Por triángulo notable:
$h = 3$

$\therefore A_{\triangle ABC} = \frac{8(3)}{2} = 12$

Clave D

9. Según el enunciado:



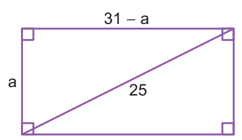
$3x + 3x + x + x = 360°$
$8x = 360°$
$x = 45°$

$A = \left(\frac{10+20}{2}\right)(5) = (15)(5) = 75$

$A = 75\ m^2$

Clave A

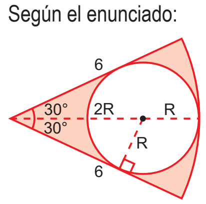
10. Del enunciado:



$(a)^2 + (31-a)^2 = 25^2 \Rightarrow a = 7$
Luego, el área es: $7 \times 24 = 168\ m^2$

Clave A

11. Según el enunciado:



$3R = 6 \Rightarrow R = 2$
Luego:

Área pedida $= A_{\triangleleft} - A_{\bigcirc}$

$= \frac{60°\pi(6)^2}{360°} - \pi(2)^2$

$= 6\pi - 4\pi = 2\pi$

Por lo tanto:
Área pedida es: $2\pi\ m^2$

Clave B

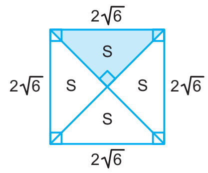
Nivel 2 (página 83) Unidad 4

Comunicación matemática

12.
13.

Razonamiento y demostración

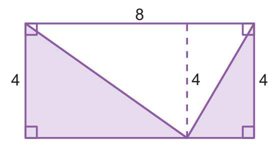
14.



$4S = (2\sqrt{6})(2\sqrt{6})$
$S = 6$

Clave E

15.



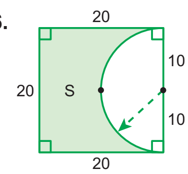
$S_{▭} = 4 \times 8 = 32$

$S_{\triangle} = \frac{4 \times 8}{2} = 16$

$\therefore S = 32 - 16 = 16$

Clave D

16.



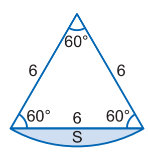
$S_{\square} = 20^2 = 400$

$S_{⌓} = \frac{\pi 10^2}{2} = 50\pi$

$S = 400 - 50\pi$
$S = 50(8 - \pi)$

Clave C

17.



$S = \frac{6^2}{2}\left(\frac{\pi 60°}{180°} - \text{sen}60°\right)$

$S = 18\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$

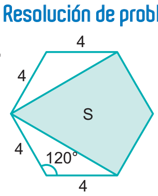
$S = 18\left(\frac{2\pi - 3\sqrt{3}}{6}\right)$

$S = 6\pi - 9\sqrt{3}$

Clave A

Resolución de problemas

18.



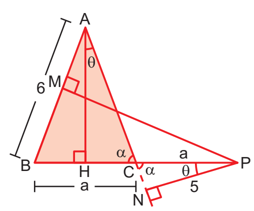
$S = \frac{6(4^2\sqrt{3})}{4} - 2\left(\frac{4(4)}{2}\text{sen}120°\right)$

$S = 24\sqrt{3} - 8\sqrt{3}$

$S = 16\sqrt{3}$

Clave B

19.



Piden:
$A_{\triangle ABC} = \frac{1}{2}a(AH)$ ...(1)

Por dato: BC = CP
Luego: ◺AHC ~ ◺PNC

$\Rightarrow \frac{AH}{5} = \frac{6}{a} \Rightarrow a(AH) = 30$

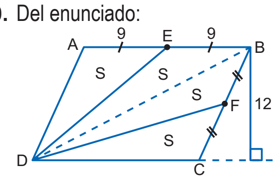
Reemplazando en (1):

$A_{\triangle ABC} = \frac{1}{2}(30) = \frac{30}{2}$

$\therefore A_{\triangle ABC} = 15$

Clave C

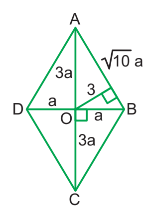
20. Del enunciado:



Área ΔABD = Área ΔDBC
Área ΔAED = Área ΔEDB
Área ΔBDF = Área ΔFDC
$\Rightarrow 4S = 18 \times 12 \Rightarrow S = 18 \times 3 = 54$
Luego, área pedida es: $2S = 108\ m^2$

Clave E

21. Del enunciado:



Del gráfico:
$(3)(\sqrt{10}\,a) = (3a)(a)$
$\Rightarrow a = \sqrt{10}$

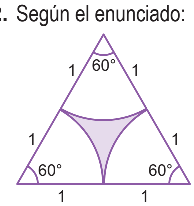
$A = \frac{(6a)(2a)}{2} = 6a^2$

$= 6(\sqrt{10})^2$

$A = 60\ cm^2$

Clave B

22. Según el enunciado:



Área pedida $= A_{\triangle} - A_{\triangleleft}$

$= \frac{(2)^2\sqrt{3}}{4} - 3\left[\frac{60°\pi(1)^2}{360°}\right]$

$= \sqrt{3} - \frac{\pi}{2}$

Por lo tanto:
Área pedida es: $\left(\sqrt{3} - \frac{\pi}{2}\right)\ m^2$

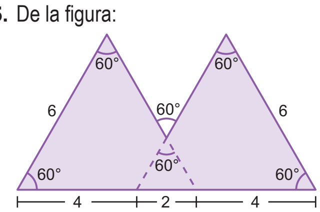
Clave D

Nivel 3 (página 83) Unidad 4

23.
24.

Razonamiento y demostración

25. De la figura:



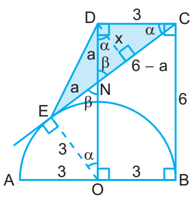
$\alpha = 60°$

$$\text{Área} = 2\left[\frac{6^2\sqrt{3}}{4} - \frac{2^2\sqrt{3}}{4}\right] + \frac{2^2\sqrt{3}}{4}$$

$$\text{Área} = 2[9\sqrt{3} - \sqrt{3}] + \sqrt{3}$$
$$\text{Área} = 2(8\sqrt{3}) + \sqrt{3} = 17\sqrt{3}\ m^2$$

Clave C

26. De la figura:



En ◺NDC:
$a^2 + 3^2 = (6 - a^2)$
$a^2 + 9 = 36 - 12a + a^2$
$12a = 27 \Rightarrow a = 2{,}25$
Además:

$x(3{,}75) = (2{,}25)(3)$

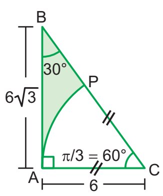
$$x = \frac{(2{,}25)(3)}{3{,}75} = 1{,}8$$

Área ΔEDC:

$$\frac{6 \times 1{,}8}{2} = 5{,}4\ m^2$$

Clave C

27.



$$S_{◺} = \frac{6 \times 6\sqrt{3}}{2} = 18\sqrt{3}$$

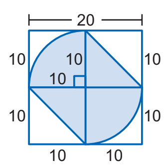
$$S_{◁} = \frac{\frac{\pi}{3} \times 6^2}{2} = 6\pi$$

$S = 18\sqrt{3} - 6\pi$

$S = 6(3\sqrt{3} - \pi)$

Clave B

28.



$$S_{◺} = \frac{10^2\pi}{4} = 25\pi$$

$$S_{◺} = \frac{10 \times 10}{2} = 50$$

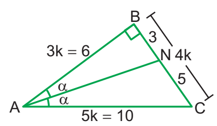
$\Sigma = 2S_{◺} + 2S_{◺}$

$\Sigma = 50\pi + 100 = 257$

Clave C

Resolución de problemas

29.

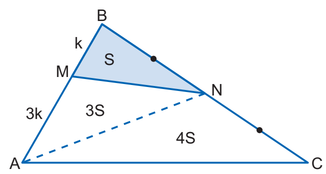


$4k = 8$
$k = 2$

$$\therefore S_{\Delta ANC} = \frac{5(6)}{2} = 15$$

Clave A

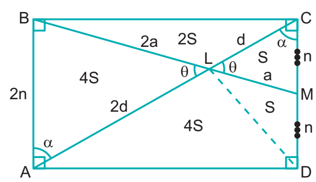
30.



Piden: $S_{\Delta MBN} = S$
Para la ceviana MN:
$S_{\Delta MNA} = 3S_{\Delta MBN} \Rightarrow S_{\Delta MNA} = 3S$
Para la mediana AN:
$S_{\Delta ANB} = S_{\Delta ANC} = 4S$
Por dato: $S_{\Delta ABC} = 64$
$\Rightarrow 8S = 64$
$\therefore S = 8$

Clave B

31.

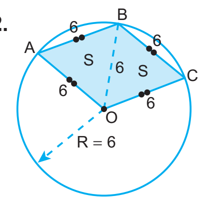


Del gráfico: ΔABL ~ ΔCML
Por dato:
$A_{▭ADML} = 5 \Rightarrow 5S = 5$
$S = 1$

Piden:
$A_{▭ABCD} = 12S = 12(1)$
$\therefore A_{▭ABCD} = 12\ m^2$

Clave D

32.



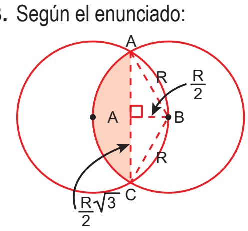
Por dato: OABC es un rombo
Del gráfico, los triángulos OAB y OBC resultan ser equiláteros.

$$\Rightarrow S = \frac{(6)^2\sqrt{3}}{4} = 9\sqrt{3}$$

Piden:
$A_{◇OABC} = 2S = 2(9\sqrt{3})$
$\therefore A_{◇OABC} = 18\sqrt{3}\ m^2$

Clave E

33. Según el enunciado:



Hallamos previamente A:
$A = A_{◁} - A_{\triangle}$

$$A = \frac{120°\pi R^2}{360°} - \frac{R}{2}\sqrt{3}\left(\frac{R}{2}\right)$$

$$A = \frac{\pi R^2}{3} - \frac{\sqrt{3}}{4}R^2$$

Piden: 2A

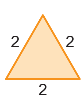
$$2A = \frac{2\pi R^2}{3} - \frac{\sqrt{3}R^2}{2} = R^2\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$$

Clave D

GEOMETRÍA DEL ESPACIO

APLICAMOS LO APRENDIDO (página 85) Unidad 4

1. $V_{prisma} = A_{base} \times h = ?$

A_{base}:  $\Rightarrow \sqrt{3}\ m^2$

$H = 10$ m
$V_{prisma} = \sqrt{3}\ m^2 \times 10\ m = 10\sqrt{3}\ m^3$

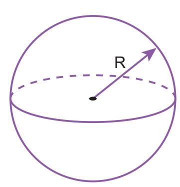
Clave C

2. Prisma cuadrangular regular:
∴ base = cuadrado
$\Rightarrow$ área de la base = 4 u × 4 u = 16 u^2

V_{prisma}: 160 u^3
$\Rightarrow V_{prisma} = \text{base (h)}$
$160\ u^3 = 16\ u^2\ (h) \Rightarrow h = 10$ u

Clave D

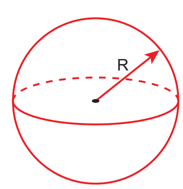
3.



$$V_{esfera} = \frac{4}{3}\pi R^3$$
$A_{esfera} = 4\pi R^2$
$\Rightarrow V_{esfera} = A_{esfera}$
$$\frac{4}{3}\pi R^3 = 4\pi R^2$$
$R = 3$

Clave C

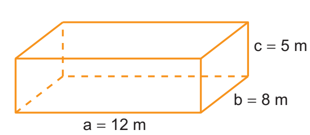
4.



$$D = 12\text{ cm} \Rightarrow R = \frac{D}{2} = \frac{12}{2} = 6\text{ cm}$$
$A = 4\pi R^2 = 4\pi(R)^2 = 144\pi\ cm^3$
$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R^3 = 288\pi\ cm^3$$

Clave D

5.

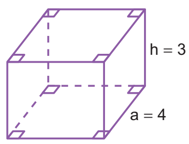


El volumen de un paralelepípedo es:
$V = abc$
Reemplazando:
$V = 12 \times 8 \times 5$
$\therefore V = 480\ m^3$

Clave A

6. Dato: $g = 4 \ \wedge\ R = 5$
Por fórmula: $A_T = 2\pi R\,(g + R)$
$\Rightarrow A_T = 2\pi(5)(4 + 5)$
$\therefore\ A_T = 90\pi$

Clave B

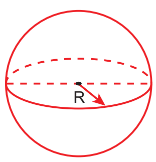
7.


En un prisma cuadrangular regular la base es un cuadrado, entonces:
$S_{base} = a^2 = (4)^2 = 16$
Nos piden el volumen:
$V = S_{base}\,h \ \Rightarrow\ V = 16(3)$
$\therefore\ V = 48$

Clave C

8. Dato:
$R = 2$
$A_{SE} = 4\pi R^2 \Rightarrow A_{SE} = 4\pi(2^2)$
$A_{SE} = 16\pi$

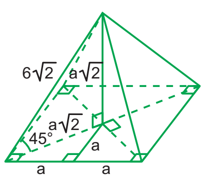
Clave B

9.


Dato: $V = 4\sqrt{3}\,\pi$
Fórmula:

$$V = \frac{4}{3}\pi R^3 \Rightarrow \frac{4}{3}\pi R^3 = 4\sqrt{3}\,\pi$$
$$R^3 = 3\sqrt{3} = \sqrt{3^3} \ \Rightarrow\ R = (\sqrt{3^3})^{\frac{1}{3}}$$
$$R = \sqrt{3}$$

Clave C

10.


$$(a\sqrt{2})\sqrt{2} = 6\sqrt{2} \Rightarrow a = 3\sqrt{2}$$
$$S_{base} = (2a)^2 = 4a^2 = 72$$
$$V = \frac{S_{base}\,h}{3} = \frac{72(3\sqrt{2})\sqrt{2}}{3}$$
$$\Rightarrow V = 144$$

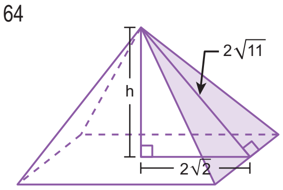
Clave D

11. Sabemos:
$A_T = 2\pi R(g + R)$ ...(1)
Del dato:
$g = h = 2x;\ R = \frac{2x}{2} = x$ y $A_T = 54\pi$
Reemplazando datos en (1):

$$54\pi = (2\pi)\frac{2x}{2}(2x + x) \ \Rightarrow\ 54 = 6x^2$$
$$9 = x^2 \qquad \therefore\ x = 3$$

Clave C

12. Nos piden el volumen: $\frac{1}{3}$ Sh
Por Pitágoras, hallamos la altura de la pirámide:
$h^2 = (2\sqrt{11})^2 - (2\sqrt{2})^2$
$h = 6$
La base es un cuadrado, entonces:
$S = (4\sqrt{2})^2 = 32$
Reemplazando:
$$V = \frac{1}{3}Sh = \frac{1}{3}(32)(6)$$
$$V = 64$$

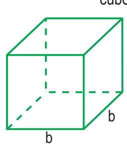


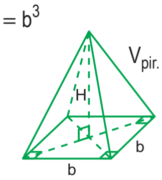
Clave C

13. Datos:
$A_L = 202{,}5\ m^2$
$A_P = 9\ m$
Sea P_{base}: semiperímetro de la base
Piden un lado: L
$A_L = P_{base}\,A_P$
$202{,}5 = P_{base}\,(9)$
$\frac{45}{2} = P_{base}$

Como es un hexágono regular = 6 lados
Entonces $P_{base} = \frac{6L}{2} = 3L$
$\frac{45}{2} = 3L \qquad \therefore L = 7{,}5\ m$

Clave E

14.
$V_{cubo} = b^3$


$V_{pir.} = \frac{1}{3}\,b^2\,h$


Por dato son equivalentes, entonces tienen el mismo volumen:
$$\underbrace{V_c} = \underbrace{V_{pir}}$$
$$b^3 = \frac{1}{3}b^2\,h \quad \Rightarrow \quad h = 3b$$

Clave A

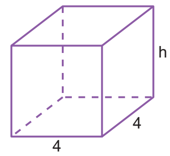
PRACTIQUEMOS

Nivel 1 (página 87) Unidad 4

Comunicación matemática

1.
2.
3.
4.

Razonamiento y demostración

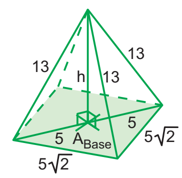
5.


$4(4)(h) = 64$
$h = 4\ m$

Clave C

6. En un cilindro se cumple:
$V = \pi R^2 h \ \Rightarrow\ 864\pi = \pi x^2 4x$
$864 = 4x^3 \Rightarrow\ 216 = x^3$
$\sqrt[3]{216} = x \quad \therefore \quad x = 6$

Clave C

7.


$A_{Base} = (5\sqrt{2})^2 = 50$
$h = 12$
Piden:
$$V = \frac{50 \times 12}{3}$$
$$V = 200$$

Clave C

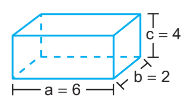
Resolución de problemas

8. Datos:
$V = 24\sqrt{3} \quad \wedge \quad a = 4\sqrt{3}$
$S_{base} = 12\sqrt{3}$
$$V = \frac{S_{base}\,h}{3}$$
Entonces:
$$24\sqrt{3} = \frac{12\sqrt{3}\ h}{3} \Rightarrow h = 6$$

Clave E

9. Dato:
$R = 3$
$$V = \frac{4\pi\,R^3}{3} \ \Rightarrow\ V = \frac{4\pi\ 27}{3}$$
$V = 36\pi$

Clave A

10.


Sabemos que en un rectoedro se cumple:
$A_T = 2(ab + ac + bc)$
Reemplazando datos:
$A_T = 2(6 \times 2 + 6 \times 4 + 2 \times 4)$
$A_T = 2(44)$
$A_T = 88$

Clave C

Nivel 2 (página 88) Unidad 4

Comunicación matemática

11.

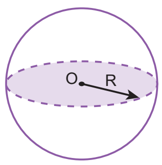
12. I. (F) porque, infinitos puntos pueden formar: recta, plano o espacio.
II. (V) por teoría.
III. (F) porque por tres puntos también podria pasar un plano.
IV. (V)porque; para el caso que los tres puntos sean colineales, sí pasaría una recta.

13. I. (F) porque la recta no tiene inicio ni fin; pero sí el rayo y el segmento tienen origen.
II. (V) porque pertenece tanto como para uno como para todas las rectas.
III. (F) porque, por un punto pasan infinitas rectas.
IV. (F) porque, para definir una recta se necesita dos puntos.

14. I. (F) porque, si fueran colineales pasan infinitos planos.
II. (V) por definición.
III. (F) porque, es "plano P" o "▱P".
IV. (F) porque, si fuesen colineales pasan infinitos planos.

Razonamiento y demostración

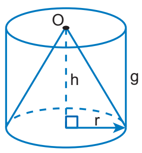
15.



$A = 4\pi R^2$
$36\pi = 4\pi R^2$
$9 = R^2 \Rightarrow R = 3$
$V = \frac{4}{3}\pi R^3 \Rightarrow V = \frac{4}{3}\pi(3)^3 = 36\pi\ m^3$

Clave C

16.



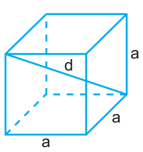
$V_{cilin.} = \pi r^2 g$ ...(I)
$V_{cono} = \frac{1}{3}\pi r^2 h$ ...(II)

Del gráfico: $g = h$
Dividiendo (I) y (II):

$$\frac{V_{cilin.}}{V_{cono}} = \frac{\pi r^2 g}{\frac{\pi r^2 g}{3}} = 3 \Rightarrow \frac{V_{cilin.}}{V_{cono}} = \frac{3}{1}$$

Clave C

17.

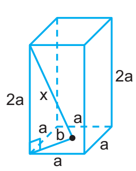


$\Rightarrow d = a\sqrt{3}$
$3\sqrt{3} = a\sqrt{3} \Rightarrow a = 3$
$A_T = 6a^2 = 6(3)^2 = 54\ m^2$

Clave B

Resolución de problemas

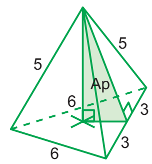
18.



Dato:
$V = 16$
$V = a^2(2a)$
$16 = a^2(2a)$
$2 = a \wedge b = \sqrt{2}$
$x^2 = (2a)^2 + b^2$
$x^2 = 18 \Rightarrow x = 3\sqrt{2}$

Clave C

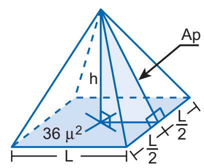
19.



$A_P = 4$
$P_{base} = \frac{6+6+6}{2} = 9$
Piden:
$A_L = 4 \times 9$
$A_L = 36$

Clave D

20.



Dato:
$A_L = 2A_{base} \Rightarrow 2L^2 = 72 \Rightarrow L = 6$
$2A_{base} = A_L \Rightarrow 2L^2 = \frac{4L(Ap)}{2}$
$L = Ap \Rightarrow h = \frac{L\sqrt{3}}{2} = 3\sqrt{3}$

Piden:
$V = \frac{36 \times 3\sqrt{3}}{3} \Rightarrow V = 36\sqrt{3}$

Clave E

Nivel 3 (página 89) Unidad 4

Comunicación matemática

21. I. (F) porque, pueden ser colineales o coplanarias.
II. (F) porque, pueden ser coplanarias.
III. (F) porque, pueden ser colineales.
IV. (V) por definición.

Clave A

22. I. (F) porque un poliedro tienen lados poligonales.
II. (V) por definición.
III. (F) porque son 5.
IV. (V) por definición.

Clave E

23. I. (F) por definición la base es un polígono.
II. (F) por definición la base es un polígono.
III. (V) por definición.
IV. (V) por definición.

Clave D

24. I. (V), por definición.
II. (V), por definición.
III. (V), el hexágono regular o cubo es un caso particular de un prisma.
IV. (V), cumple la definición de prisma.

Razonamiento y demostración

25. Por dato: $\pi R^2 = 81\pi \Rightarrow R = 9$
$S_{esfera} = 4\pi R^2 = 4\pi(9)^2 = 324\pi$
$V_{esfera} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(9)^3 = 972\pi$

Clave D

26. $A_T = 54\pi$
$A_T = A_L + 2A_{base}$
$A_L = 2\pi\left(\frac{x}{2}\right)x = \pi x^2$
$A_{base} = \frac{\pi x^2}{4} \Rightarrow 54\pi = \pi x^2 + 2\left(\frac{\pi x^2}{4}\right)$
$54 = x^2 + \frac{x^2}{2} \Rightarrow 54 = \frac{3x^2}{2} \Rightarrow x = 6$

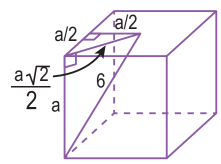
Clave A

27. El hecho de que dos sólidos sean equivalentes implica que sus volúmenes son iguales.
$V_{esfera} = \frac{4}{3}\pi x^3$
$V_{cono} = \frac{1}{3}\pi x^2(4)$
$\Rightarrow \frac{4}{3}\pi x^3 = \frac{4\pi x^2}{3} \Rightarrow x^3 = x^2 \Rightarrow x = 1$

Clave A

Resolución de problemas

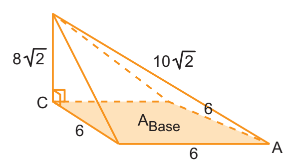
28.



$6^2 = a^2 + \left(\frac{a}{2}\sqrt{2}\right)^2$
$36 = a^2 + \frac{a^2}{2} = \frac{3a^2}{2} \Rightarrow a = 2\sqrt{6}$
$V = a^3 = (2\sqrt{6})^3 \Rightarrow V = 48\sqrt{6}$

Clave B

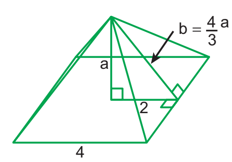
29.



Obs.: $AC = 6\sqrt{2}$
$A_{base} = 6^2 = 36$
Piden V:
$V = \frac{36 \times 8\sqrt{2}}{3} \Rightarrow V = 96\sqrt{2}$

Clave D

30.



$A_{L_1} = 4\left(\frac{4b}{2}\right) \Rightarrow A_{L_1} = 8b$

Dato:
$A_{L_1} = \frac{2}{3}A_{L_2}$
$8b = \frac{2}{3}(16a) \Rightarrow b = \frac{4}{3}a$

Por Pitágoras:
$a^2 + 2^2 = \left(\frac{4a}{3}\right)^2 \Rightarrow a = \frac{6\sqrt{7}}{7}$

Clave D

APLICAMOS LO APRENDIDO (página 90) Unidad 4

1. Piden: $P' = \text{Sim } (-5; -3)_{(Q)}$
$P' = \text{Sim } (-5; -3)_{(Q)}$
Luego:

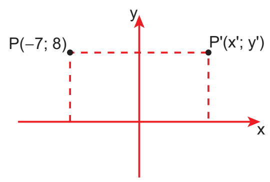
$\Rightarrow Q(-1; -1)\left\{\begin{aligned} x_0 &= -1 \\ y_0 &= -1 \end{aligned}\right.$

$\Rightarrow P(-5; -3)\left\{\begin{aligned} x &= -5 \\ y &= -3 \end{aligned}\right.$

Hallamos las coordenadas de P':
$x' = 2(-1) - (-5) \Rightarrow x' = 3$
$y' = 2(-1) - (-3) \Rightarrow y' = 1 \Rightarrow P'(3; 1)$

Clave A

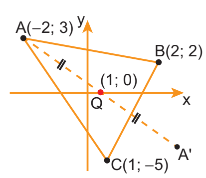
2. Graficamos P:



Piden:
$P' = \text{Sim } P_{(y)}$
$P' = \text{Sim } (-7; 8)_{(y)}$; $\overline{y}$ es eje de simetría
$x = -7$
$y = 8$
Coordenadas de P'(x'; y'):
$x' = -(-7)$ $\quad y' = (8)$
$\Rightarrow P'(7; 8)$; luego $x' + y' = 15$

Clave D

3.

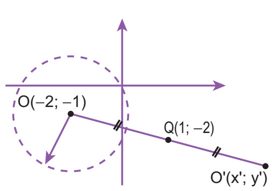


$A' = \text{Sim } A_{(Q)}$
$A' = \text{Sim } (-2; 3)_{(Q)}$
Luego:
$Q(1; 0)\left\{\begin{aligned} x_0 &= 1 \\ y_0 &= 0 \end{aligned}\right. \qquad A(-2; 3)\left\{\begin{aligned} x &= -2 \\ y &= 3 \end{aligned}\right.$

Coordenadas de A'(x', y'):
$x' = 2(1) - (-2) = 4$
$y' = 2(0) - (3) = -3$
$\therefore A'(4; -3)$

Clave E

4.

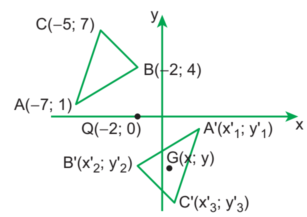


$O' = \text{Sim } O_{(a)}$
$O' = \text{Sim } (-2; -1)_{(Q)}$
Luego:
$Q(1; -2)\left\{\begin{aligned} x_0 &= 1 \\ y_0 &= -2 \end{aligned}\right. \qquad O(-2; -1)\left\{\begin{aligned} x &= -2 \\ y &= -1 \end{aligned}\right.$

$x' = 2(1) - (-2) = 4$
$y' = 2(-2) - (-1) = -3$
$\Rightarrow O'(4; -3)$

Clave D

5. Graficamos el plano cartesiano:



Si Q(−2; 0) es el punto de simetría
$\Rightarrow$ Se cumple
AQ = QA'
BQ = QB'
CQ = QC'
$\Rightarrow A' = \text{Sim } A_{(Q)}$
$(x_1'; y_1') = \text{Sim } (x_1; y_1)_{(Q)}$
$(x_1'; y_1') = \text{Sim } (-7; 1)_{(Q)}$
$Q(-2; 0) = (x_0; y_0)$

$x_1' = 2x_0 - x_1 \qquad y_1' = 2y_0 - x_1$
$x_1' = 2(-2) - (-7) \qquad y_1' = 2(0) - 1$
$x_1' = 3 \qquad y_1' = -1$
$\Rightarrow A'(3; -1)$

$\Rightarrow B' = \text{Sim } B_{(Q)}$
$(x_2'; y_2') = \text{Sim } (x_2; y_2)_{(Q)}$
$(x_2'; y_2') = \text{Sim } (-2; 4)_{(Q)}$
$Q(-2; 0) = (x_0; y_0)$

$x_2' = 2x_0 - x_2 \qquad y_2' = 2y_0 - y_2$
$x_2' = 2(-2) - (-2) \qquad y_2' = 2(0) - (4)$
$x_2' = -2 \qquad y_2' = -4$
$\Rightarrow B'(-2; -4)$

$\Rightarrow C' = \text{Sim } C_{(Q)}$
$(x_3'; y_3') = \text{Sim } (x_3; y_3)_{(Q)}$
$(x_3'; y_3') = \text{Sim } (-5; 7)_{(Q)}$
$Q(-2; 0) = (x_0; y_0)$

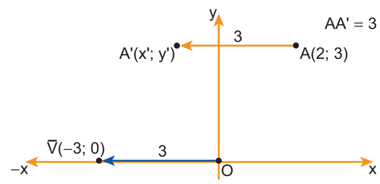
$x_3' = 2x_0 - x_3 \qquad y_3' = 2y_0 - x_3$
$x_3' = 2(-2) - (-5) \qquad y_3' = 2(0) - (7)$
$x_3' = 1 \qquad y_3' = -7$
$C'(1; -7)$

Tenemos los 3 vértices del nuevo triángulo A'B'C', por lo tanto podemos hallar su baricentro G(x; y)

$x = \frac{1}{3}(x_1' + y_2' + x_3')$
$\Rightarrow x = \frac{1}{3}(3 + (-2) + 1) \Rightarrow x = 2/3$
$y = \frac{1}{3}(y_1' + y_2' + y_3')$
$\Rightarrow y = \frac{1}{3}((-1) + (-4) + (-7)) \Rightarrow y = -4$
$\therefore G = (2/3; -4)$

Clave C

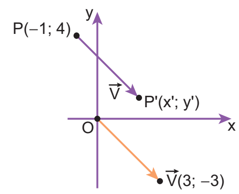
6. Graficamos el plano cartesiano:



$\Rightarrow A' = \text{Tras } A_{(V)}$; $V(-3; 0)$
$A' = \text{Tras } (2; 3)_{(V)}$; $x_0 = -3$
Hallamos coordenadas de A':
$\left.\begin{aligned} x' &= (2) + (-3) \Rightarrow x' = 2 - 3 \Rightarrow x' = -1 \\ y' &= (3) \qquad\quad \Rightarrow y' = 3 \quad\;\; \Rightarrow y' = 3 \end{aligned}\right\} A'(-1; 3)$

Clave A

7. Ubicamos $\vec{V}(3; -3)$ en el plano cartesiano



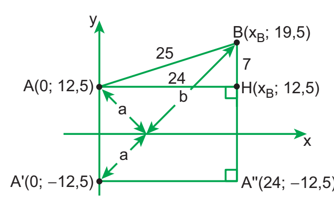
$\Rightarrow P' = \text{Tras } P_{(\vec{V})}$; $\vec{V}(3; -3)$
$P' = \text{Tras } (-1; 4)_{(\vec{V})}$ $\qquad x_0 = 3;\; y_0 = -3$
Coordenadas de P':
$x' = (-1) + (3) \qquad y' = (4) + (-3)$
$x' = 2 \qquad y' = 1$
$\Rightarrow P'(2; 1)$

Clave A

8. $A' = (-2; 3) + V$
$A' = (-2; 3) + (2; -5)$
$A' = (0; -2)$

Clave B

9. Graficamos el plano cartesiano:



Aplicamos el teorema del Pitágoras en el ◺ABH
$25^2 = x_B^2 + 7^2$
$x_B^2 = 25^2 - 7^2 \Rightarrow x_B^2 = (25 + 7)(25 - 7)$

$x_B^2 = 32 \times 18 \Rightarrow x_B = 8 \times 3 = 24$

Aplicamos simetría:

$\Rightarrow$ A' = Sim $A_{(x)}$

A' = Sim $(0; 12{,}5)_{(x)} = (0; -12{,}5)$

Aplicamos traslación

$\Rightarrow$ A'' = Tras $A'_{(24;\,0)}$

A'' = Tras $(0; -12{,}5)_{(24;\,0)}$

A'' = (24; −12,5)

Luego distancia mínima del recorrido = a + b

$d_{min} = a + b$

Aplicamos el teorema de Pitágoras en el ⊿A'A''B

$(a + b)^2 = 24^2 + (12{,}5 + 12{,}5 + 7)^2$

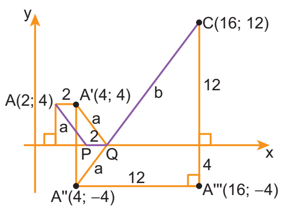
$d^2 = 24^2 + 32^2$

$d^2 = 40^2$

$\Rightarrow d = 40$

Clave C

10. Graficamos el plano cartesiano



Piden: AP + QC

$$\left.\begin{aligned} AP &= a \\ QC &= b \end{aligned}\right\} +$$

$AP + QC = a + b$

$x = a + b$ ... (I)

Hacemos la traslación:

A' = Tras $A_{(2;\,0)}$

A' = Tras $(2; 4)_{(2;\,0)} \Rightarrow$ A' = (4; 4)

y luego aplicamos simetría:

A'' = Sim $A'_{(\bar{x})}$

A'' = Sim $(4; 4)_{(\bar{x})}$

A'' = (4; −4)

del gráfico de deduce que el recorrido

APQC < > AA'QC = 2 + a + b ... (II)

$\Rightarrow$ hacemos la traslación:

A''' = Tras $A''_{(12;\,0)}$

A''' = Tras $(4; -4)_{(12;\,0)} \Rightarrow$ A''' = (16; −4)

En el A''A'''C aplicamos el teorema de Pitágoras:

$(a + b)^2 = 12^2 + 16^2$

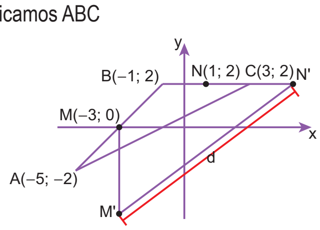
de (I): $x^2 = 12^2 + 16^2$

$x^2 = 20^2$

$x = 20$

Clave D

11. Graficamos ABC



Hallamos $M(x_1; y_1)$

$$\left.\begin{aligned} x_1 &= \frac{1-5}{2} \Rightarrow x_1 = -3 \\ y_1 &= \frac{2-2}{2} \Rightarrow y_1 = 0 \end{aligned}\right\} M(-3; 0)$$

$\Rightarrow$ traslación de $M(x_1; y_1)$ en dirección $\bar{y}$

M' = $\text{TrasM}_{(0;\,-4)}$

$(x_1'; y_1') = \text{Tras}(x_1; y_1)_{(0;\,4)}$

$(x_1'; y_1') = \text{Tras}(-3; 0)_{(0;\,4)}$

Coordenadas de M':

$x_1' = x_1 + x_0 \qquad x_1' = y_1 - y_0$

$x_1' = -3 + 0 \qquad x_1' = 0 - 4$

$\therefore$ M'(−3; −4)

Hallamos $N(x_2; y_2)$

$$\left.\begin{aligned} x_2 &= \left(-\frac{1+3}{2}\right) \Rightarrow x_2 = 1 \\ y_2 &= \left(\frac{2+2}{2}\right) \Rightarrow y_2 = 2 \end{aligned}\right\} N(1; 2)$$

$\Rightarrow$ traslación de $N(x_2; y_2)$ en dirección $+\bar{x}$

N' = Tras $N_{(4;\,0)}$

$(x_2'; y_2') = \text{Tras}(x_2; y_2)_{(4;\,0)}$

$(x_2'; y_2') = \text{Tras}(1; 2)_{(4;\,0)}$

Coordenadas de N'

$x_2' = x_2 + x_0 \qquad y_2' = y_2 + y_0$

$x_2' = 1 + 4 \qquad y_2' = 2 + 0$

$\therefore$ N'(5; 2)

Como tenemos M'(−3; 4) y N'(5; 2) podemos calcular la distancia que los separa

$d = \sqrt{(x_2' - y_1')^2 + (y_2' - y_1')^2}$

$d = \sqrt{(5-(-3))^2 + (2-(-4))^2}$

$d = 10$

Clave A

12. B' = Rot $B_{(O;\,90°)}$

B' = Rot $(5; -3)_{(O;\,90°)}$

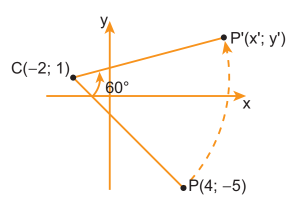
Coordenadas de B'

$x' = -(-3) \wedge y' = 5$

$\Rightarrow$ B' $\Rightarrow$ B' = (3; 5) $\therefore$ 5 − 3 = 2

Clave D

13. Ubicamos el punto (4; −5)



Piden: P' = Rot $P_{(C;\,60°)}$

P' = Rot $(4; -5)_{(C;\,60°)}$; C(−2; 1)

$x = 4 \qquad x_0 = -2$

$y = -5 \qquad y_0 = 1$

Coordenadas de P'(x'; y'):

$x' = (-2) + (4 - (-2))\cos 60° - (-5 - (1))\text{sen}60°$

$x' = -2 + 6\left(\frac{1}{2}\right) + 6\frac{\sqrt{3}}{2}$

$y' = 1 + (4 - (-2))\text{sen}60° + (-5 - 1)\cos 60°$

$y' = 1 + (6)\frac{\sqrt{3}}{2} + (-6)\frac{1}{2}$

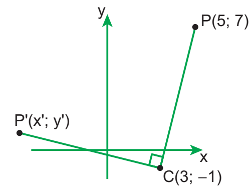
$\therefore P'(3\sqrt{3} + 1; 3\sqrt{3} - 2)$;

luego $x' - y' = 3\sqrt{3} + 1 - 3\sqrt{3} + 2$

$x' - y' = 3$

Clave C

14.



Piden: P' = Rot $P_{(C;\,90°)}$

P' = Rot $(5; 7)_{(C;\,90°)}$; C(3; −1)

$x = 5 \qquad x_0 = 3$

$y = 7 \qquad y_0 = -1$

Coordenadas de P'(x'; y')

$\begin{aligned} x' &= (3) - (7) + (-1) \\ y' &= (-1) + (5) - (3) \end{aligned} \Rightarrow \begin{aligned} x' &= -5 \\ y' &= 1 \end{aligned}$

$\therefore$ P'(−5; 1); luego x'y' = −5

Clave B

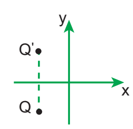
PRACTIQUEMOS

Nivel 1 (página 92) Unidad 4

Comunicación matemática

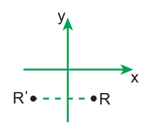
1.

2. I.



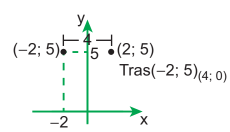
(F)

II.



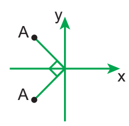
(F)

III.



(V)

IV.



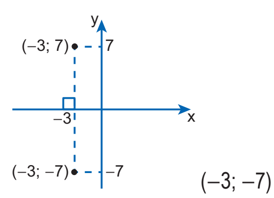
(V)

Rot $A_{(0,90)} \in$ IIIC

Clave D

Razonamiento y demostración

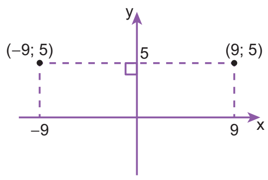
3.



(−3; −7)

Clave C

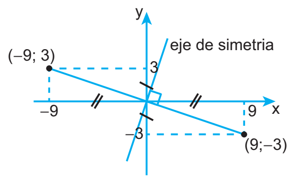
4.



(9, 5)

Clave E

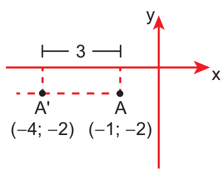
5.



x' – y' = (9) – (–3)
x' – y' = 12

Clave B

6.

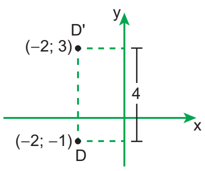


∴ A'(–4; –2)

Clave B

Resolución de problemas

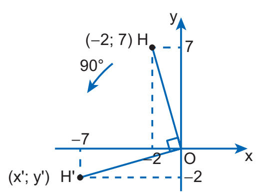
7.



∴ D'(–2; 3)

Clave E

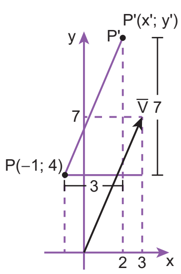
8.



x' = –(7)
y' = (–2)
∴ H' = (–7, –2)

Clave A

9.

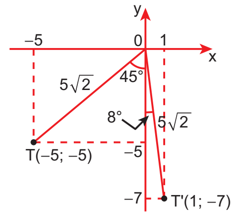


P' = Tras $P_{(\vec{V})}$

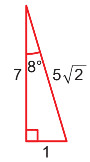
P' = Tras $(-1; 4)_{(3;\,7)}$
P' = (–1 + 3; 4 + 7)
P' = (2; 11)
∴ x' + y' = 13

Clave C

10.



En el gráfico:



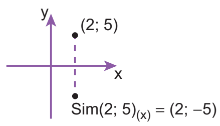
Piden: 1 + (–7) = –6

Clave B

Nivel 2 (página 92) Unidad 4

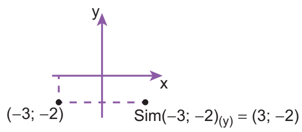
Comunicación matemática

11. I.



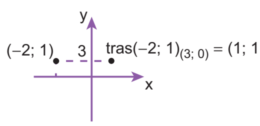
(F)

II.



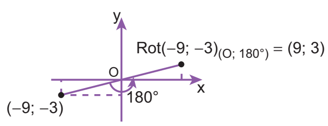
(V)

III.



(F)

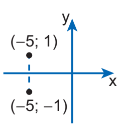
IV.



(V)

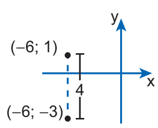
Clave A

12. I.



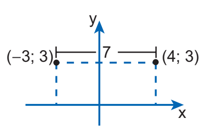
(V)

II.



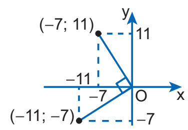
(V)

III.



(V)

IV.

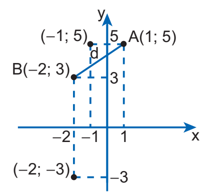


(F)

Clave C

Razonamiento y demostración

13.

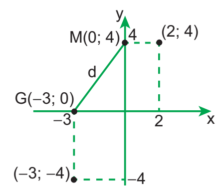


$d = \sqrt{(-2-1)^2 + (3-5)^2}$

$d = \sqrt{9+4} = \sqrt{13}$

Clave B

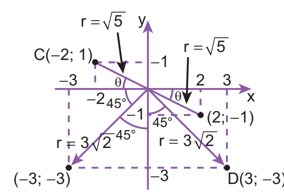
14.



$d = \sqrt{(0-(-3))^2 + (4-0)^2} = 5$

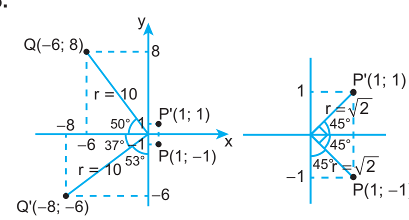
Clave E

15.



Clave E

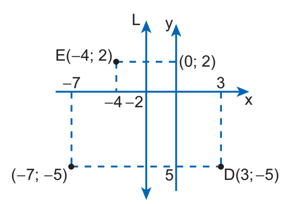
16.



Clave A

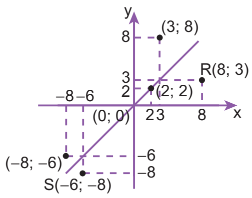
Resolución de problemas

17.



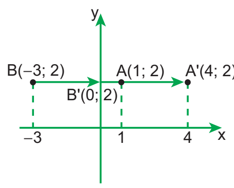
Clave C

18.



Clave A

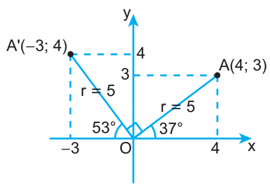
19.

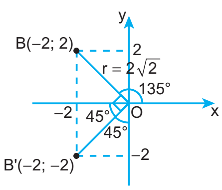


$A'(1 + 3; 2) = A'(4; 2)$
$B'(-3 + 3; 2) = B'(0; 2)$

Clave C

20.



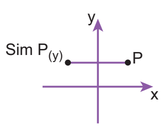


Clave B

Nivel 3 (página 93) Unidad 4

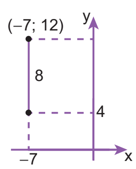
Comunicación matemática

21. I.



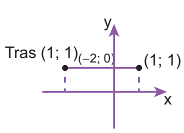
(V)

II.



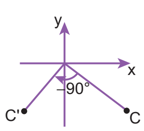
(F)

III.



(V)

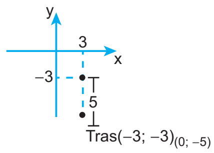
IV.



(F)

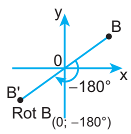
Clave B

22. I.



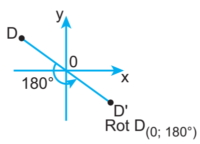
(F)

II.



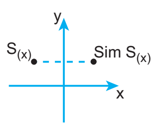
(F)

III.



(V)

IV.

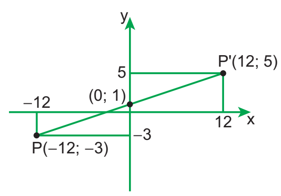


(V)

Clave E

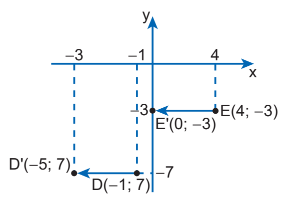
Razonamiento y demostración

23.



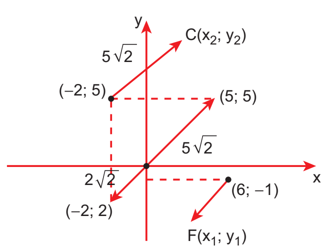
Clave B

24.



Clave A

25.

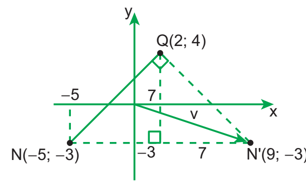


Coordenadas de F:
$x_1 = 6 + (-2) = 4 \Rightarrow F = (4; -3)$
$y_1 = -1 + (-2) = -3 \Rightarrow F = (4; -3)$
Coordenadas de C:
$x_2 = -2 + (5) = 3$
$y_2 = 5 + (5) = 10 \Rightarrow C = (3; 10)$

Clave A

Resolución de problemas

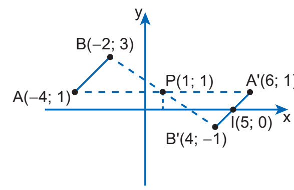
26.



$v^2 = 9^2 + 3^2 \Rightarrow v^2 = 3^2(3^2 + 1) \Rightarrow v = 3\sqrt{10}$

Clave A

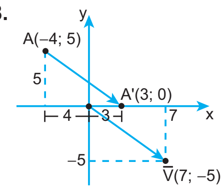
27.



I: es el punto de intersección
X_r: abscisa de I $\Rightarrow X_r = 5$

Clave E

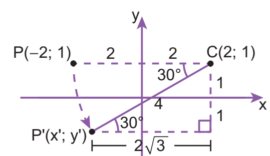
28.



Coordenadas de A'
$\Rightarrow 3 = x_1 + 7$
$x_1 = -4$
$\Rightarrow 0 = y_1 + (-5)$
$y_1 = 5$
$\therefore A = (-4;5)$
$\Rightarrow -4 + 5 = 1$

Clave D

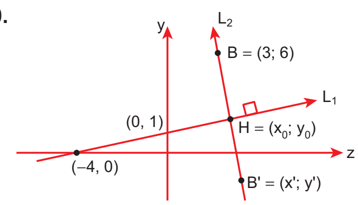
29.



$P' = (-(2\sqrt{3} - 2); -1)$
$P' = (2 - 2\sqrt{3}; -1)$
$\text{prod} = 2\sqrt{3} - 2$

Clave C

30.

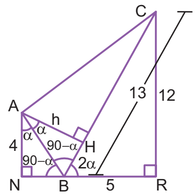


$L_1: y = \frac{x}{4} + 1$
L_2: $y = -4x + a$; pero $B \in L_2 \Rightarrow 6 = -4(3) + a$
$\Rightarrow a = 18$
L_2: $y = -4x + 18$
Luego $H \in L_1 \wedge H \in L_2$ (Igualando $L_1 = L_2$)
$\frac{x_0}{4} + 1 = -4x_0 + 18 \Rightarrow x_0 = 4 \wedge y_0 = z \;\therefore\; H(4; 2)$
Finalmente: $B' = \text{Sim } B_{(H)} \Rightarrow B' = \text{Sim}(3:6)_{(4,2)}$
Coordenadas de B':
$x' = 2(4) - 3 \;\wedge\; y' = 2(2) - 6$
$x' = 5 \;\wedge\; y' = -2 \;\Rightarrow B' = (5; -2)$
$\therefore V_B = \sqrt{5^2 + (-2)^2} \Rightarrow V_B = \sqrt{29}$

Clave B

MARATÓN MATEMÁTICA (página 94)

1. Por el teorema de Pitágoras.



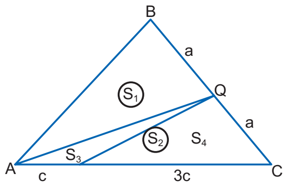
$\triangle$ CRB:
$BC^2 = BR^2 + CR^2$
$BC = 13$
Por congruencia de triángulos:
$\triangle ANB \cong \triangle AHB$
$h = 4$

$\therefore A_{\triangle ABC} = \dfrac{4 \times 13}{2} = 26$

Clave C

2. Para este problema solo se usan relaciones entre áreas de relaciones triangulares.

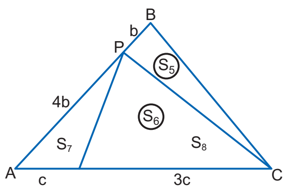
1.ª relación: en el triángulo ABC.



$S_1 = S_2 \Rightarrow S_2 = 60\ cm^2$

$\dfrac{S_3}{S_4} = \dfrac{1c}{3c} \Rightarrow \begin{cases} S3 = 15cm^2 \\ S4 = 45cm^2 \end{cases}$

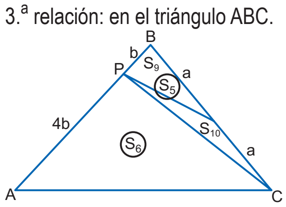
2.ª relación: en el triángulo ABC.



$\dfrac{S_5}{S_6} = \dfrac{1b}{4b} \Rightarrow \begin{matrix} S_5 = 24\ cm^2 \\ S_6 = 96\ cm^2 \end{matrix}$

$\dfrac{S_7}{S_8} = \dfrac{1c}{4c} \Rightarrow \begin{matrix} S_7 + S_8 = S_6 \\ S_7 = 24\ cm^2 \\ S_8 = 72\ cm^2 \end{matrix}$

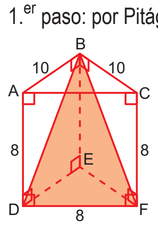
3.ª relación: en el triángulo ABC.



$S_9 = S_{10} \Rightarrow S_9 + S_{10} = S_5$
$S_9 = 12\ cm^2$
$\therefore A_{\triangle PQR} = A_{\triangle ABC} - S_4 - S_7 - S_9 = 39\ cm^2$

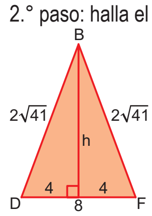
Clave D

3. 1.^{er} paso: por Pitágoras;



$DB^2 = AD^2 + AB^2$
$\Rightarrow DB = 2\sqrt{41}$
Por semejanza
$DB = BF = 2\sqrt{41}$

2.° paso: halla el área $\triangle DBF$;

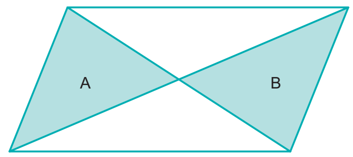


Por Pitágoras $h = 2\sqrt{37}$

$\therefore A_{\triangle DBF} = \dfrac{h(DF)}{2}$
$= 8\sqrt{37}$

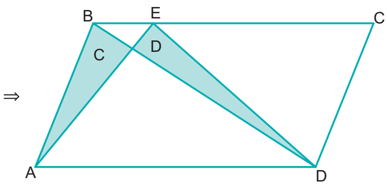
Clave A

4. Por relaciones de áreas en un trapecio:



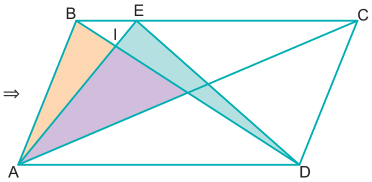
$\mathbb{A} = \mathbb{B}$

$\Rightarrow$



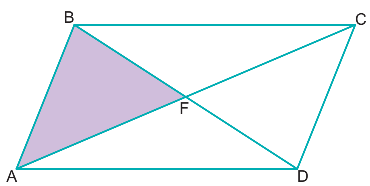
$\mathbb{C} = \mathbb{D}$

$\Rightarrow$



En la figura se observa que $A_{\triangle IED} = A_{\triangle BIA}$ por relaciones de áreas en un trapecio.

$\therefore$



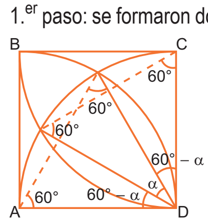
Por relaciones de áreas en un paralelogramo:

$A_{\triangle ABF} = \dfrac{1}{4} A_{\square ABCD}$

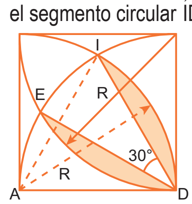
$\dfrac{Área \triangle ABF}{Área \square ABCD} = \dfrac{1}{4}$

Clave C

5. 1.^{er} paso: se formaron dos triángulos equiláteros.



2.° paso: calculamos las áreas sombreadas para el segmento circular $\overset{\frown}{ID}$.

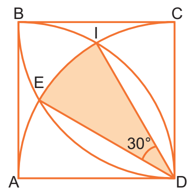


$Área\ \overset{\frown}{ID} = \dfrac{60°\pi R^2}{360°} - \dfrac{R^2 sen60°}{2}$

$Área\ \overset{\frown}{ID} = 4\left(\dfrac{2}{3}\pi - \sqrt{3}\right)$

$Área\ \overset{\frown}{ID} = Área\ \overset{\frown}{DE} \Rightarrow A_{sombreada} = 8\left(\dfrac{2}{3}\pi - \sqrt{3}\right)$

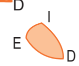
3.° paso: hallamos el área del sector circular $\overset{\frown}{EDI}$.

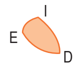


$A_{\triangleleft EDI} = \pi 4^2 \dfrac{30°}{360°}$

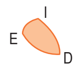
$A_{\triangleleft EDI} = \dfrac{4}{3}\pi$

$\therefore$ El área pedida





$= \dfrac{4}{6}\pi + 8\left(\dfrac{2}{3}\pi - \sqrt{3}\right) = \dfrac{20\pi}{3} - 8\sqrt{3}$

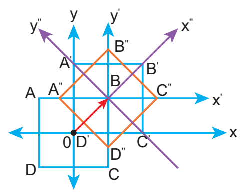


$= 4\left(\dfrac{5\pi}{3} - 2\sqrt{3}\right)$

Clave D

6. 1.^{er} paso: trasladar el punto O al punto B;

$\Rightarrow$ se le suma el vector $OB = (2; 2)$.



Respecto a "O":
$A = (-2; 2) \Rightarrow A' = (-2; 2) + (2; 2) = (0; 4)$
$B = (2; 2) \Rightarrow B' = (2; 2) + (2; 2) = (4; 4)$
$C = (2; -2) \Rightarrow C' = (2; -2) + (2; 2) = (4; 0)$
$D = (-2; -2) \Rightarrow D' = (-2; -2) + (2; 2) = (0; 0)$

2.° paso: rotar 45°;
recuerda: $x" = x'\cos\theta - y'sen\theta$
$y" = x'sen\theta + y'\cos\theta$

$45°$ y $\cos 45° = sen45° = \dfrac{\sqrt{2}}{2}$

Respecto a "B":

$A" \begin{cases} x" = -2\left(\dfrac{\sqrt{2}}{2}\right) - (2)\left(\dfrac{\sqrt{2}}{2}\right) = 2\sqrt{2} \\ y" = (-2)\left(\dfrac{\sqrt{2}}{2}\right) + (2)\left(\dfrac{\sqrt{2}}{2}\right) = 0 \end{cases}$

$A" = (-2\sqrt{2}, 0)$

$D" = \begin{cases} x" = (-2)\left(\dfrac{\sqrt{2}}{2}\right) - (-2)\left(\dfrac{\sqrt{2}}{2}\right) = 0 \\ y" = (-2)\left(\dfrac{\sqrt{2}}{2}\right) + (-2)\left(\dfrac{\sqrt{2}}{2}\right) = -2\sqrt{2} \end{cases}$

$D" = (0, -2\sqrt{2})$

3.^{er} paso: hallamos A" y D" respecto de "O":
$A" = (-2\sqrt{2}, 0) + (2, 2) = (2 - 2\sqrt{2}, 2)$
$D" = (0, -2\sqrt{2}) + (2, 2) = (2, 2 - 2\sqrt{2})$

$\therefore$ El $A_{\triangle A"BD"} = \dfrac{|A" - B||D" - B|}{2}$

$A_{\triangle A"BO"} = \dfrac{|(2-2\sqrt{2}, 2) - (2, 2)||(2, 2-2\sqrt{2}) - (2, 2)|}{2}$

$A_{\triangle A"BO"} = \dfrac{(2\sqrt{2})(2\sqrt{2})}{2} = 4$

Clave D